Subexponential Parameterized Algorithms for Bounded-Degree Connected Subgraph Problems on Planar Graphs

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The *bidimensionality* theory [2] provides a general framework to obtain subexponential parameterized algorithms for a broad family of NP-hard problems on planar graphs (and more generally, for *sparse* families of graphs). The aim of this talk is to illustrate this methodology with the following parameterized problem (for $d \geq 2$) :

MAXIMUM d-DEGREE-BOUNDED CONNECTED SUBGRAPH (MDBCS_d)
Input : A graph G and a non-negative integer k.
Parameter : k.
Question : Does G contain a connected subgraph H with $\Delta(H) \leq d$ and $|E(H)| \geq k$?

Notice that if $d = 2$, MDBCS_d is equivalent to the LONGEST PATH (or CYCLE, if G is Hamiltonian) problem and can be seen as a generalisation of it. This problem is one of the classical NP-hard problems listed by Garey and Johnson in [3], and it has been recently proved in [1] that MDBCS_d is not in APX for any $d \geq 2$. It turns out that without the connectivity constraint, this problem is known to be solvable in polynomial time (for any d) using matching techniques [4].

Our target is to obtain a $2^{\mathcal{O}(\sqrt{k})} \cdot \mathcal{O}(n)$ step algorithm (with $n = |V(G)|$) for MDBCS_d when the input is restricted to planar graphs. The same strategy can be extended to several problems asking for a maximum connected subgraph satisfying certain degree constraints. Loosely speaking, the idea is as follows. We define the following parameter on simple undirected graphs :

$$\mathbf{medbcs}_d(G) = \max\{|E(H)| \ \mid \ H \subseteq G \ \wedge \ H \text{ is connected } \ \wedge \ \Delta(H) \leq d\}.$$

Then we distinguish two cases according to the *branchwidth* of G ($\mathbf{bw}(G)$ for short) :

- If $\mathbf{bw}(G)$ is *big* (greater than $\sqrt{k}$), we must exhibit a *certificate* that $\mathbf{medbcs}_d(G)$ is also *big* (proving that the parameter is *minor-closed* and looking at its behaviour on the square grid), and then the answer to the parameterized problem is automatically YES.
- Otherwise, if $\mathbf{bw}(G)$ is *small* (smaller than $\sqrt{k}$), we compute $\mathbf{medbcs}_d(G)$ efficiently using Catalan structures and dynamic programming techniques over an optimal branch decomposition of G.

References

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