Dynamic programming for graphs on surfaces

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[An extended abstract appeared in **ICALP'10**]

Outline

1. Background
2. Motivation and previous work
3. Main ideas of our approach
4. Sketch of the enumerative part
5. Conclusions and further research

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Branch decompositions and branchwidth

- A branch decomposition of a graph $G = (V, E)$ is tuple (T, μ) where:
  - T is a tree where all the internal nodes have degree 3.
  - μ is a bijection between the leaves of T and $E(G)$.
- Each edge $e \in T$ partitions $E(G)$ into two sets A_e and B_e.
- For each $e \in E(T)$, we define $\mathbf{mid}(e) = V(A_e) \cap V(B_e)$.
- The width of a branch decomposition is $\max_{e \in E(T)} |\mathbf{mid}(e)|$.
- The branchwidth of a graph G (denoted $\mathbf{bw}(G)$) is the minimum width over all branch decompositions of G:

$$\mathbf{bw}(G) = \min_{(T,\mu)} \max_{e \in E(T)} |\mathbf{mid}(e)|$$

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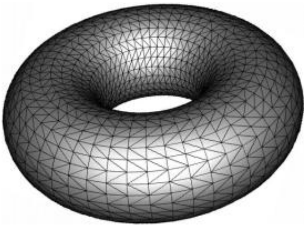
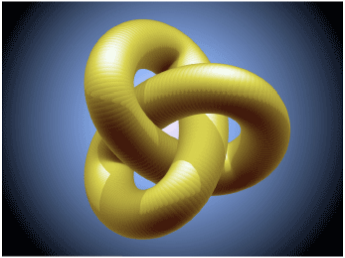
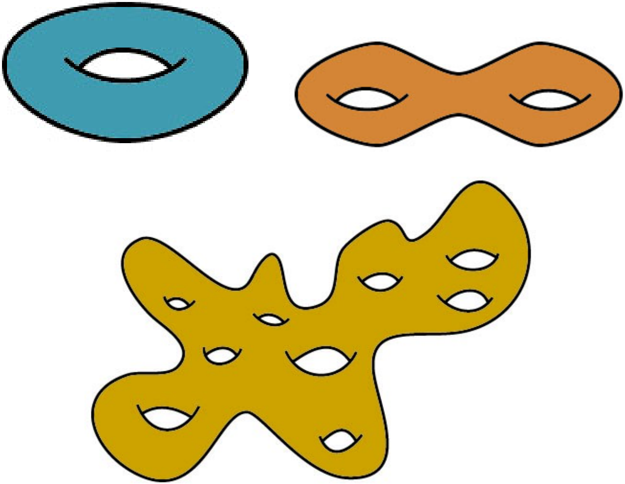
Surfaces

- **SURFACE** = TOPOLOGICAL SPACE, LOCALLY "FLAT"

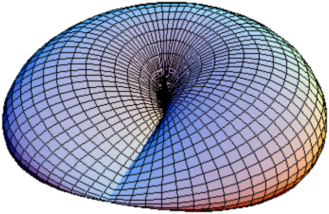


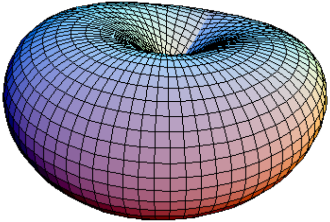


Handles



Cross-caps





Surface Classification Theorem

- **Surface Classification Theorem**:

  any compact, connected and without boundary surface can be obtained from the sphere $\mathbb{S}^2$ by adding handles and cross-caps.

- Orientable surfaces:

  obtained by adding $g \geq 0$ *handles* to the sphere $\mathbb{S}^2$, obtaining the g-torus $\mathbb{T}_g$ with Euler genus $\mathbf{eg}(\mathbb{T}_g) = 2g$.

- Non-orientable surfaces:

  obtained by adding $h > 0$ *cross-caps* to the sphere $\mathbb{S}^2$, obtaining a non-orientable surface $\mathbb{P}_h$ with Euler genus $\mathbf{eg}(\mathbb{P}_h) = h$.

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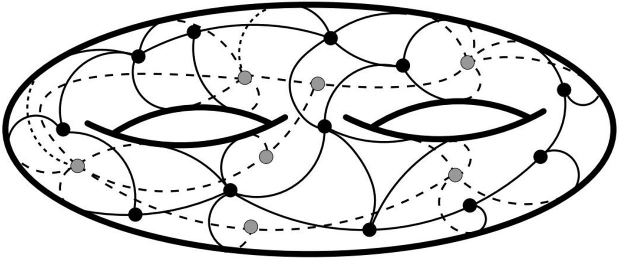
Graphs on surfaces

EMBEDDED GRAPH: GRAPH DRAWN ON A SURFACE, NO CROSSINGS



- The **Euler genus of a graph** G, $\mathbf{eg}(G)$, is the least Euler genus of the surfaces in which G can be embedded.

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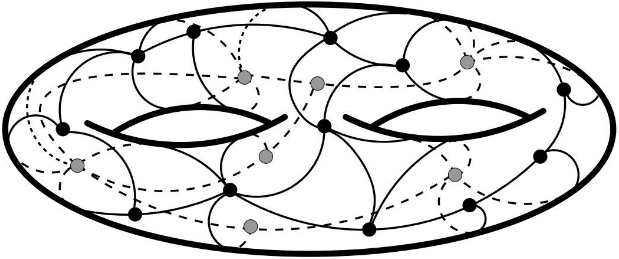

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Some words on parameterized complexity

- Idea: given an NP-hard problem, fix one parameter of the input to see if the problem gets more "tractable".

  Example: the size of a VERTEX COVER.

- Given a (NP-hard) problem with input of size n and a parameter k, a fixed-parameter tractable (FPT) algorithm runs in

$$f(k) \cdot n^{\mathcal{O}(1)}, \quad \text{for some function } f.$$

Examples: k-VERTEX COVER, k-LONGEST PATH.

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FPT and single-exponential algorithms

- Courcelle's theorem (1988):

  Graph problems expressible in Monadic Second Order Logic (MSOL) can be solved in time $f(k) \cdot n^{\mathcal{O}(1)}$ in graphs G such that $\mathbf{bw}(G) \leq k$.

- **Problem**: $f(k)$ can be huge!!! (for instance, $f(k) = 2^{3^{4^{5^{6^{k}}}}}$)

- A single-exponential parameterized algorithm is a FPT algo s.t.

$$f(k) = 2^{\mathcal{O}(k)}.$$

Objective: build a framework to obtain **single-exponential parameterized algorithms** for a class of NP-hard problems in **graphs embedded on surfaces.**

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- Applied in a bottom-up fashion on a rooted branch decomposition of the input graph G.
- For each graph problem, DP requires the suitable definition of tables encoding how potential (global) solutions are restricted to a middle set $\mathbf{mid}(e)$.
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A classification of graph optimization problems

How can we certificate a solution in a middle set **mid**(e)?

1. A subset of vertices of **mid**(e) (not restricted by some global condition).
   Examples: VERTEX COVER, DOMINATING SET.
   The size of the tables is bounded by $2^{\mathcal{O}(k)}$.

2. A *connected pairing* of vertices of **mid**(e).
   Examples: LONGEST PATH, CYCLE PACKING, HAMILTONIAN CYCLE.
   The # of pairings in a set of k elements is $k^{\Theta(k)} = 2^{\Theta(k \log k)}$...

   Done for planar graphs [Dorn, Penninkx, Bodlaender, Fomin. *ESA'05*];
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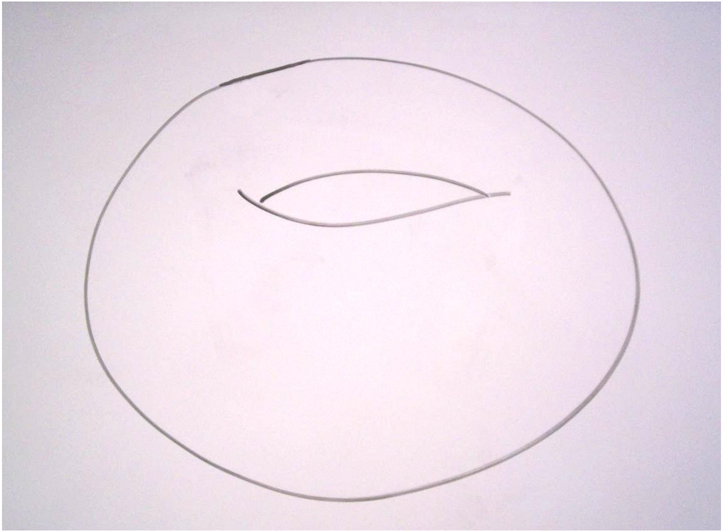

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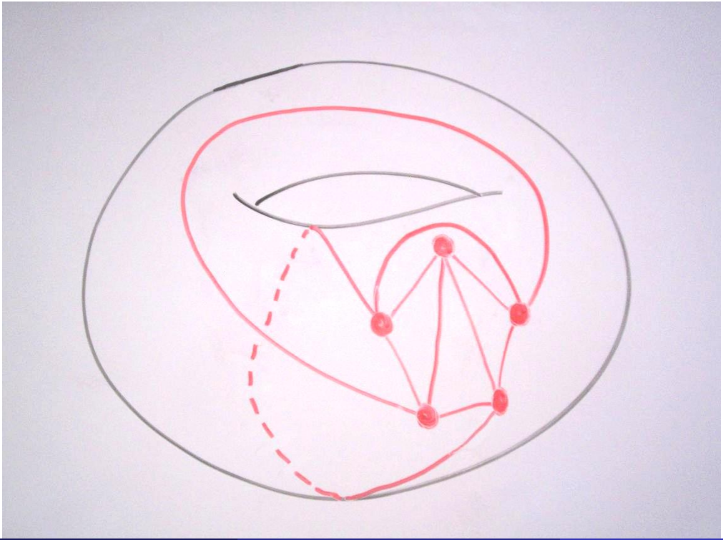

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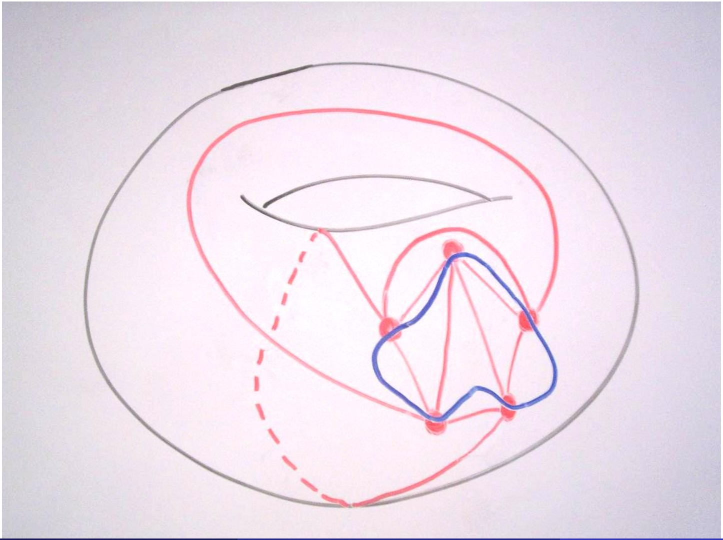

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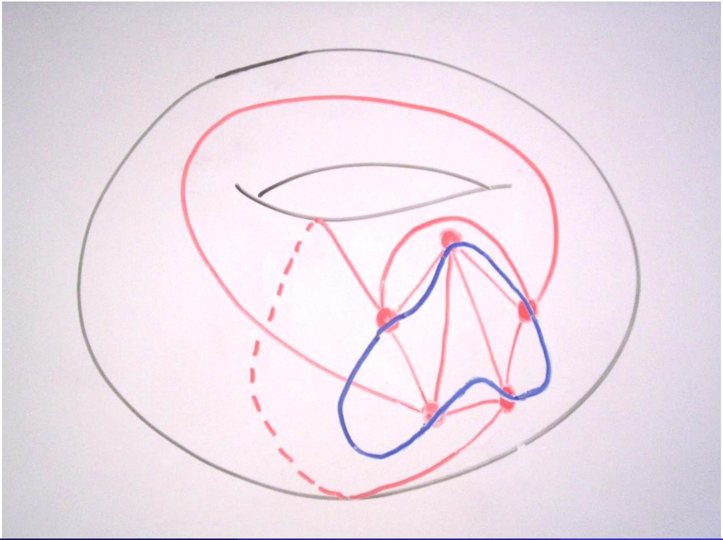

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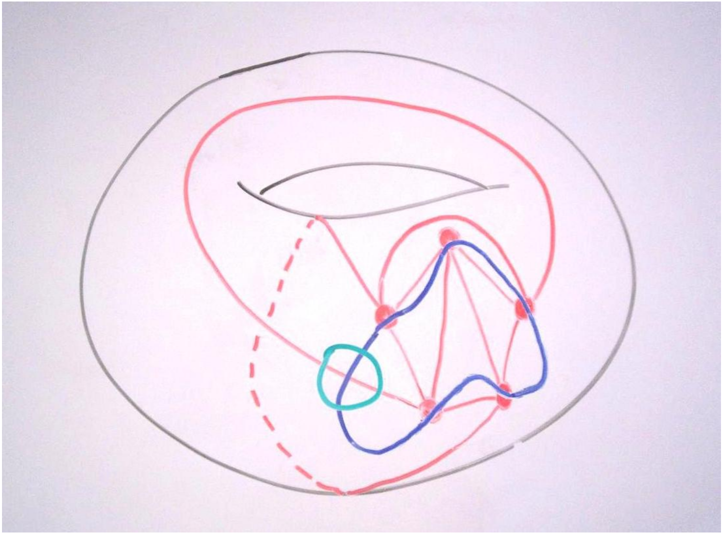

Sphere cut decompositions

Key idea for planar graphs [Dorn *et al.* *ESA'05*]:

- Sphere cut decomposition: Branch decomposition where the vertices in each **mid**(e) are situated around a noose.
  [Seymour and Thomas. *Combinatorica'94*]

- Recall that the size of the tables of a DP algorithm depends on how many ways a partial solution can intersect **mid**(e).
- In how many ways can we draw polygons inside a circle such that they touch the circle only on its k vertices and they do not intersect?

- Exactly the number of *non-crossing partitions* over k elements, which is given by the k-th Catalan number:

$$CN(k) = \frac{1}{k+1}\binom{2k}{k} \sim \frac{4^k}{\sqrt{\pi}k^{3/2}} \approx 4^k.$$

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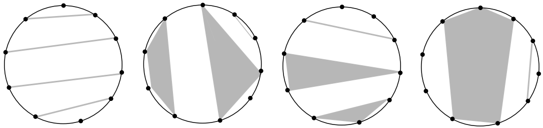

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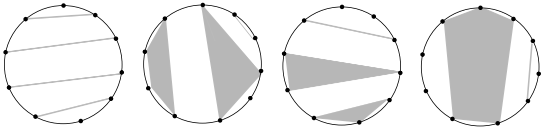

- Exactly the number of *non-crossing partitions* over k elements, which is given by the k-th Catalan number:

$$\mathrm{CN}(k) = \frac{1}{k+1}\binom{2k}{k} \sim \frac{4^k}{\sqrt{\pi}k^{3/2}} \approx 4^k.$$

“Old” idea for graphs on surfaces

Key idea for graphs on surfaces [Dorn *et al.* *SWAT’06*]:

- Perform a planarization of the input graph by splitting the potential solutions into a number of pieces depending on the surface.
- Then, apply the sphere cut decomposition technique to a more complicated version of the problem where the number of pairings is still bounded by some Catalan number.
- Drawbacks of this technique:
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From sphere to surface cut decompositions

Our approach is based on a new type of branch decomposition, called surface cut decomposition.

- Surface cut decompositions for graphs on surfaces **generalize** sphere cut decompositions for planar graphs. [Seymour and Thomas. *Combinatorica'94*]

- That is, we exploit directly the combinatorial structure of the potential solutions in the surface (**without planarization**).

- Using surface cut decompositions, we provide in a **unified** way single-exponential algorithms for **connected packing-encodable** problems, and with **better genus** dependence.

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Surface cut decompositions (simplified version)

Let G be a graph embedded in a surface Σ, with $\mathbf{eg}(\Sigma) = \mathbf{g}$.

A surface cut decomposition of G is a branch decomposition (T, μ) of G and a subset $A \subseteq V(G)$, with $|A| = \mathcal{O}(\mathbf{g})$, s.t. for all $e \in E(T)$

- either $|\mathbf{mid}(e) \setminus A| \leq 2$,
- or
  - ⋆ the vertices in $\mathbf{mid}(e) \setminus A$ are contained in a set $\mathcal{N}$ of $\mathcal{O}(\mathbf{g})$ nooses;
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Surface cut decompositions can be efficiently computed:

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Sketch of the construction of surface cut decompositions:

- Partition G into **polyhedral** pieces, plus a set of A vertices, with $|A| = O(\mathbf{g})$.
- For each piece H, compute a branch decomposition, using Amir's algorithm.
- Transform this branch decomposition to a **carving** decomposition of the **medial** graph of H.
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- This fact is proved using **analytic combinatorics**, generalizing Catalan structures to arbitrary surfaces.
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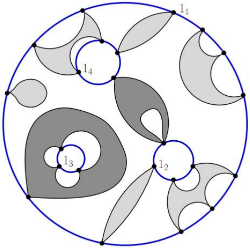
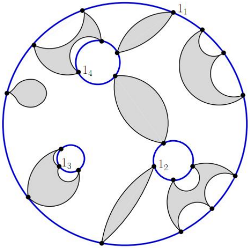
Bipartite subdivisions

- Subdivision of the surface in vertices, edges and **2-dimensional regions** (not necessary contractible).
- All vertices lay in the boundary.
- 2 types of 2-dimensional regions: **black** and **white**.
- Each vertex is incident with exactly 1 black region (also called *block*).
- Each border is rooted.

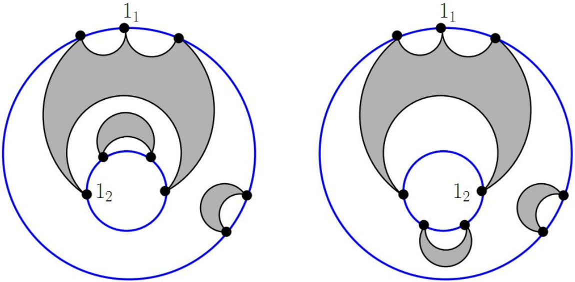




Fixing the number of vertices on a given surface, we have an infinite number of bipartite subdivisions.

Non-crossing partitions in higher genus surfaces

- Each bipartite subdivision induces a non-crossing partition on the set of vertices.
- **Problem:** Different bipartite subdivisions can define the same non-crossing partition.



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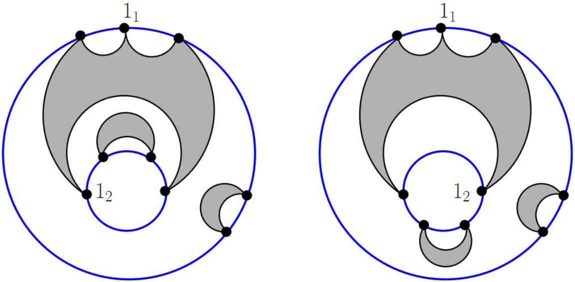

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The strategy

We make the problem "easier" by reducing it to a **map enumeration** problem:

1. For each bipartite subdivision there exists another bipartite subdivision, with all the blocks **contractible**, with the same associated non-crossing partition.
2. We show that the greatest contribution comes from bipartite subdivisions where white faces are **contractible**.
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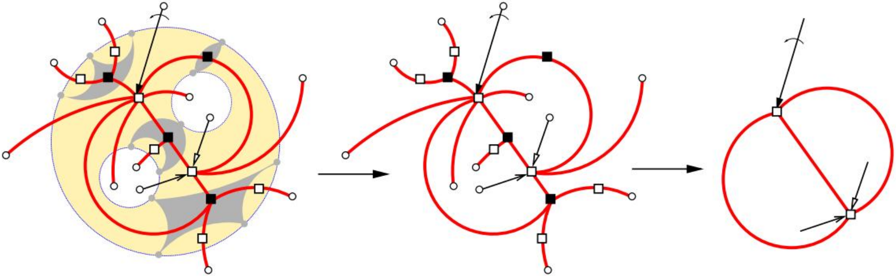
The enumeration (I)

We exploit the ideas used to asymptotically count simplicial decompositions on surfaces with boundaries [Bernardi, Rué. *Manuscript'09*]:



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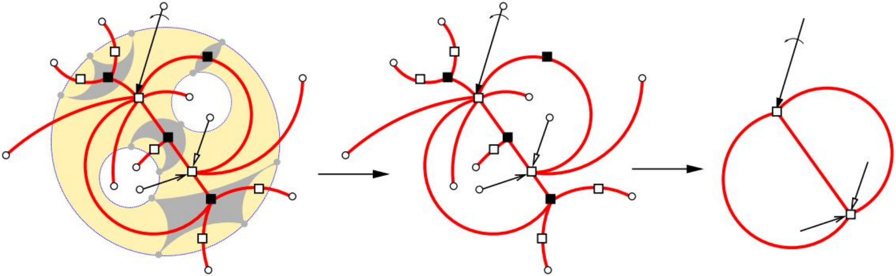

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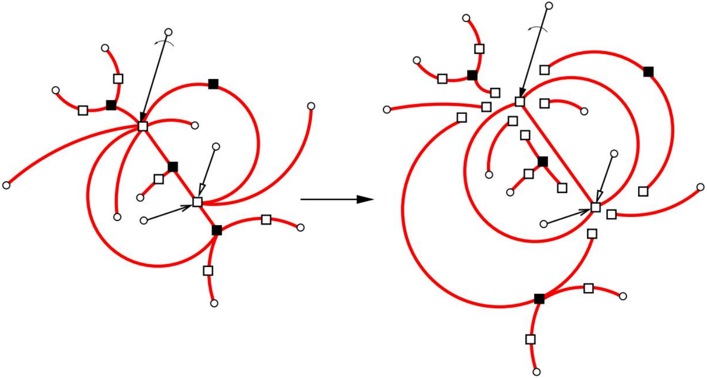

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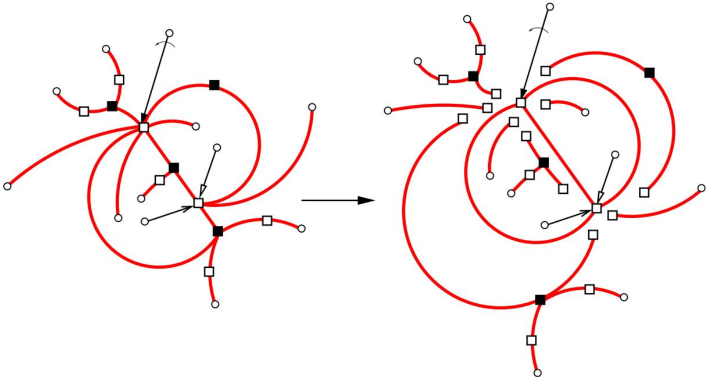

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Main enumerative result

After some study of bicolored trees and its asymptotics...

Theorem (Rué, Thilikos, S.)

Let Σ be a surface with boundary. Then the number of non-crossing partitions on Σ with k vertices is asymptotically bounded by

$$\frac{C(\Sigma)}{\Gamma\left(-3/2\chi(\Sigma)+\beta(\Sigma)\right)} \cdot k^{-3/2\chi(\Sigma)+\beta(\Sigma)-1} \cdot 4^k \cdot (1+o(1)),$$

where

- $C(\Sigma)$ is a function depending only on Σ (cubic maps in $\overline{\Sigma}$ with $\beta(\Sigma)$ faces).
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- We presented a framework for the design of DP algorithms on **surface-embedded** graphs running in time $2^{\mathcal{O}(k)} \cdot n$.
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  1. Let **P** be a **connected packing-encodable** problem on a surface-embedded graph G.
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Further research

1. Improve the constants in the running times.
2. Can this framework be applied to **more complicated problems**?

   Fundamental problem: H-minor containment

   - ⋆ Minor containment for host graphs G on surfaces.
     [Adler, Dorn, Fomin, S., Thilikos. *SWAT'10*]

     With running time $2^{\mathcal{O}(k)} \cdot h^{2k} \cdot 2^{\mathcal{O}(h)} \cdot n$.

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     Truly single-exponential: $2^{\mathcal{O}(h)} \cdot n$.

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Further research

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Gràcies!