On the complexity of finding large odd induced subgraphs and odd colorings

Ignasi Sau

LIRMM, Université de Montpellier, CNRS, Montpellier, France

Joint work with **Rémy Belmonte**

University of Electro-Communications, Chofu, Japan

virtUMA 2020

21-25 septiembre 2020

Outline of the talk

1. Introduction
2. Our results
3. Some proofs
4. Further research

Next section is...

A graph is even/odd if all its vertex degrees are even/odd.

A graph is even/odd if all its vertex degrees are even/odd.

Theorem (Gallai ∼1960)

For every graph G, $V(G)$ can be partitioned into two sets

- V_1 *and* V_2 *such that both* $G[V_1]$ *and* $G[V_2]$ *are even,*

A graph is even/odd if all its vertex degrees are even/odd.

Theorem (Gallai ∼1960)

For every graph G, $V(G)$ can be partitioned into two sets

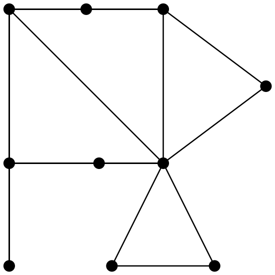
- V_1 *and* V_2 *such that both* $G[V_1]$ *and* $G[V_2]$ *are even, and*
- V_1' *and* V_2' *such that* $G[V_1']$ *is even and* $G[V_2']$ *is odd.*

A graph is even/odd if all its vertex degrees are even/odd.

Theorem (Gallai ∼1960)

For every graph G, $V(G)$ can be partitioned into two sets

- V_1 *and* V_2 *such that both* $G[V_1]$ *and* $G[V_2]$ *are even, and*
- V_1' *and* V_2' *such that* $G[V_1']$ *is even and* $G[V_2']$ *is odd.*

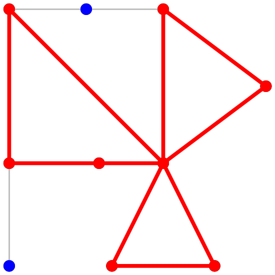


A graph is even/odd if all its vertex degrees are even/odd.

Theorem (Gallai ∼1960)

For every graph G, $V(G)$ can be partitioned into two sets

- V_1 *and* V_2 *such that both* $G[V_1]$ *and* $G[V_2]$ *are even, and*
- V_1' *and* V_2' *such that* $G[V_1']$ *is even and* $G[V_2']$ *is odd.*

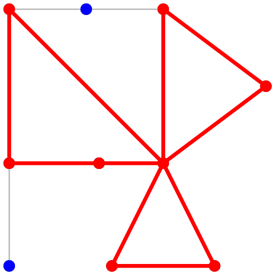


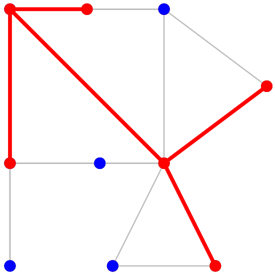
A graph is even/odd if all its vertex degrees are even/odd.

Theorem (Gallai ∼1960)

For every graph G, $V(G)$ can be partitioned into two sets

- *V_1 and V_2 such that both $G[V_1]$ and $G[V_2]$ are even, and*
- *V_1' and V_2' such that $G[V_1']$ is even and $G[V_2']$ is odd.*



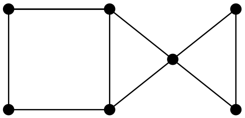


A graph is even/odd if all its vertex degrees are even/odd.

Theorem (Gallai ∼1960)

For every graph G, $V(G)$ can be partitioned into two sets

- *V_1 and V_2 such that both $G[V_1]$ and $G[V_2]$ are even, and*
- *V_1' and V_2' such that $G[V_1']$ is even and $G[V_2']$ is odd.*



A graph is even/odd if all its vertex degrees are even/odd.

Theorem (Gallai ∼1960)

For every graph G, $V(G)$ can be partitioned into two sets

- V_1 *and* V_2 *such that both* $G[V_1]$ *and* $G[V_2]$ *are even, and*
- V_1' *and* V_2' *such that* $G[V_1']$ *is even and* $G[V_2']$ *is odd.*

A graph is even/odd if all its vertex degrees are even/odd.

Theorem (Gallai ∼1960)

For every graph G, $V(G)$ can be partitioned into two sets

- V_1 *and* V_2 *such that both* $G[V_1]$ *and* $G[V_2]$ *are even, and*
- V_1' *and* V_2' *such that* $G[V_1']$ *is even and* $G[V_2']$ *is odd.*

Corollary

Every graph G contains an even induced subgraph with at least $|V(G)|/2$ vertices.

Are similar properties true for odd subgraphs?

Are similar properties true for odd subgraphs?

For a graph G, let

$\mathrm{mos}(G)$: order of a largest odd induced subgraph of G.

Are similar properties true for odd subgraphs?

For a graph G, let

$\mathrm{mos}(G)$: order of a largest odd induced subgraph of G.

$\chi_{\mathrm{odd}}(G)$: minimum number of odd induced subgraphs of G that partition $V(G)$.

Are similar properties true for odd subgraphs?

For a graph G, let

$\mathrm{mos}(G)$: order of a largest odd induced subgraph of G.

$\chi_{\mathrm{odd}}(G)$: minimum number of odd induced subgraphs of G that partition $V(G)$.

For $\chi_{\mathrm{odd}}(G)$ to be well-defined, each connected component of G must have even order.

Are similar properties true for odd subgraphs?

For a graph G, let

$\mathrm{mos}(G)$: order of a largest odd induced subgraph of G.

$\chi_{\mathrm{odd}}(G)$: minimum number of odd induced subgraphs of G that partition $V(G)$.

For $\chi_{\mathrm{odd}}(G)$ to be well-defined, each connected component of G must have even order.

$\mathrm{mes}(G)$ and $\chi_{\mathrm{even}}(G)$: symmetric parameters for the even version.

Are similar properties true for odd subgraphs?

For a graph G, let

$\mathrm{mos}(G)$: order of a largest odd induced subgraph of G.

$\chi_{\mathrm{odd}}(G)$: minimum number of odd induced subgraphs of G that partition $V(G)$.

For $\chi_{\mathrm{odd}}(G)$ to be well-defined, each connected component of G must have even order.

$\mathrm{mes}(G)$ and $\chi_{\mathrm{even}}(G)$: symmetric parameters for the even version.

Hence, for every graph G on n vertices: $\mathrm{mes}(G) \geq n/2$ and $\chi_{\mathrm{even}}(G) \leq 2$.

Are similar properties true for odd subgraphs?

For a graph G, let

$\mathrm{mos}(G)$: order of a largest odd induced subgraph of G.

$\chi_{\mathrm{odd}}(G)$: minimum number of odd induced subgraphs of G that partition $V(G)$.

For $\chi_{\mathrm{odd}}(G)$ to be well-defined, each connected component of G must have even order.

$\mathrm{mes}(G)$ and $\chi_{\mathrm{even}}(G)$: symmetric parameters for the even version.

Hence, for every graph G on n vertices: $\mathrm{mes}(G) \geq n/2$ and $\chi_{\mathrm{even}}(G) \leq 2$.

$\boxed{\text{What about } \mathrm{mos}(G) \text{ and } \chi_{\mathrm{odd}}(G)\text{?}}$

“Part of the graph theory folklore” [Caro. 1994]

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\mathrm{mos}(G) \geq c \cdot n$.

“Part of the graph theory folklore” [Caro. 1994]

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\mathrm{mos}(G) \geq c \cdot n$.

- For every “good” G, $\mathrm{mos}(G) \geq (1 - o(1))\sqrt{n/6}$. [Caro. 1994]

"Part of the graph theory folklore" [Caro. 1994]

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\mathrm{mos}(G) \geq c \cdot n$.

- For every "good" G, $\mathrm{mos}(G) \geq (1 - o(1))\sqrt{n/6}$. [Caro. 1994]
- For every "good" G, $\mathrm{mos}(G) \geq \frac{cn}{\log n}$ for some $c > 0$. [Scott. 1992]

"Part of the graph theory folklore" [Caro. 1994]

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\mathrm{mos}(G) \geq c \cdot n$.

- For every "good" G, $\mathrm{mos}(G) \geq (1 - o(1))\sqrt{n/6}$. [Caro. 1994]
- For every "good" G, $\mathrm{mos}(G) \geq \frac{cn}{\log n}$ for some $c > 0$. [Scott. 1992]
- The conjecture has been proved for particular graph classes:
  - Trees. [Radcliffe, Scott. 1995]
  - Graphs G with bounded $\chi(G)$. [Scott. 1992]
  - Graphs G with $\Delta(G) \leq 3$. [Berman, Wang, Wargo. 1997]
  - Graphs G with $\mathrm{tw}(G) \leq 2$. [Hou, Yu, Li, Liu. 2018]

"Part of the graph theory folklore" [Caro. 1994]

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\mathrm{mos}(G) \geq c \cdot n$.

- For every "good" G, $\mathrm{mos}(G) \geq (1 - o(1))\sqrt{n/6}$. [Caro. 1994]
- For every "good" G, $\mathrm{mos}(G) \geq \frac{cn}{\log n}$ for some $c > 0$. [Scott. 1992]
- The conjecture has been proved for particular graph classes:
  - Trees. [Radcliffe, Scott. 1995]
  - Graphs G with bounded $\chi(G)$. [Scott. 1992]
  - Graphs G with $\Delta(G) \leq 3$. [Berman, Wang, Wargo. 1997]
  - Graphs G with $\mathrm{tw}(G) \leq 2$. [Hou, Yu, Li, Liu. 2018]

In these articles, they obtain best possible constants $c > 0$.

"Part of the graph theory folklore" [Caro. 1994]

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\mathrm{mos}(G) \geq c \cdot n$.

- For every "good" G, $\mathrm{mos}(G) \geq (1 - o(1))\sqrt{n/6}$. [Caro. 1994]
- For every "good" G, $\mathrm{mos}(G) \geq \frac{cn}{\log n}$ for some $c > 0$. [Scott. 1992]
- The conjecture has been proved for particular graph classes:
  - Trees. [Radcliffe, Scott. 1995]
  - Graphs G with bounded $\chi(G)$. [Scott. 1992]
  - Graphs G with $\Delta(G) \leq 3$. [Berman, Wang, Wargo. 1997]
  - Graphs G with $\mathrm{tw}(G) \leq 2$. [Hou, Yu, Li, Liu. 2018]

In these articles, they obtain best possible constants $c > 0$.

The conjecture is still open.

$\boxed{\text{What about } \chi_{\text{odd}}(G)\text{?}}$

What about $\chi_{\text{odd}}(G)$?

For $\chi_{\text{odd}}(G)$ to be well-defined, each connected component of G must have even order.

$\boxed{\text{What about } \chi_{\text{odd}}(G)\text{?}}$

For $\chi_{\text{odd}}(G)$ to be well-defined, each connected component of G must have even order.

This necessary condition is also sufficient. [Scott. 2001]

What about $\chi_{\text{odd}}(G)$?

For $\chi_{\text{odd}}(G)$ to be well-defined, each connected component of G must have even order.

This necessary condition is also sufficient. [Scott. 2001]

Upper and lower bounds on $\chi_{\text{odd}}(G)$: [Scott. 2001]

What about $\chi_{\text{odd}}(G)$?

For $\chi_{\text{odd}}(G)$ to be well-defined, each connected component of G must have even order.

This necessary condition is also sufficient. [Scott. 2001]

Upper and lower bounds on $\chi_{\text{odd}}(G)$: [Scott. 2001]

- For every "good" graph G on n vertices,

$$\chi_{\text{odd}}(G) \le \frac{cn}{\sqrt{\log \log n}} = o(n).$$

What about $\chi_{\text{odd}}(G)$?

For $\chi_{\text{odd}}(G)$ to be well-defined, each connected component of G must have even order.

This necessary condition is also sufficient. [Scott. 2001]

Upper and lower bounds on $\chi_{\text{odd}}(G)$: [Scott. 2001]

- For every "good" graph G on n vertices,

$$\chi_{\text{odd}}(G) \leq \frac{cn}{\sqrt{\log \log n}} = o(n).$$

- There are "good" graphs G on n vertices for which

$$\chi_{\text{odd}}(G) \geq (1 + o(1))\sqrt{2n} = \Omega(\sqrt{n}).$$

What about $\chi_{\text{odd}}(G)$?

For $\chi_{\text{odd}}(G)$ to be well-defined, each connected component of G must have even order.

This necessary condition is also sufficient. [Scott. 2001]

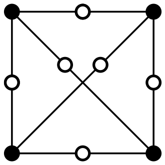
Upper and lower bounds on $\chi_{\text{odd}}(G)$: [Scott. 2001]

- For every "good" graph G on n vertices,

$$\chi_{\text{odd}}(G) \leq \frac{cn}{\sqrt{\log \log n}} = o(n).$$

- There are "good" graphs G on n vertices for which

$$\chi_{\text{odd}}(G) \geq (1 + o(1))\sqrt{2n} = \Omega(\sqrt{n}).$$



G = subdivided n-clique with $n \equiv 0, 3 \pmod 4$

What about $\chi_{\text{odd}}(G)$?

For $\chi_{\text{odd}}(G)$ to be well-defined, each connected component of G must have even order.

This necessary condition is also sufficient. [Scott. 2001]

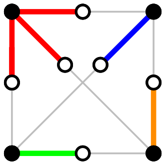
Upper and lower bounds on $\chi_{\text{odd}}(G)$: [Scott. 2001]

- For every "good" graph G on n vertices,

$$\chi_{\text{odd}}(G) \leq \frac{cn}{\sqrt{\log \log n}} = o(n).$$

- There are "good" graphs G on n vertices for which

$$\chi_{\text{odd}}(G) \geq (1 + o(1))\sqrt{2n} = \Omega(\sqrt{n}).$$



G = subdivided n-clique with $n \equiv 0, 3 \pmod 4$

$\boxed{\text{And what about the complexity of computing these parameters?}}$

$\boxed{\text{And what about the complexity of computing these parameters?}}$

Computing $\mathrm{mes}(G)$ and $\mathrm{mos}(G)$ is NP-hard. [Cai, Yang. 2011]

$\boxed{\text{And what about the complexity of computing these parameters?}}$

Computing $\text{mes}(G)$ and $\text{mos}(G)$ is NP-hard. [Cai, Yang. 2011]

For every graph G, $\chi_{\text{even}}(G) \leq 2$, so it is easy.

$\boxed{\text{And what about the complexity of computing these parameters?}}$

Computing $\mathrm{mes}(G)$ and $\mathrm{mos}(G)$ is NP-hard. [Cai, Yang. 2011]

For every graph G, $\chi_{\mathrm{even}}(G) \leq 2$, so it is easy.

As for $\chi_{\mathrm{odd}}(G)$, no complexity results were known so far.

And what about the complexity of computing these parameters?

Computing $\mathrm{mes}(G)$ and $\mathrm{mos}(G)$ is NP-hard. [Cai, Yang. 2011]

For every graph G, $\chi_{\mathrm{even}}(G) \leq 2$, so it is easy.

As for $\chi_{\mathrm{odd}}(G)$, no complexity results were known so far.

Our goal Computational aspects of the parameters mos and χ_{odd}.

Next section is...

For an integer $q \geq 1$, we prove that deciding whether $\chi_{\mathsf{odd}}(G) \leq q$ is

For an integer $q \geq 1$, we prove that deciding whether $\chi_{\mathsf{odd}}(G) \leq q$ is

- polynomial-time solvable if $q \leq 2$, and
- NP-complete if $q \geq 3$.

For an integer $q \geq 1$, we prove that deciding whether $\chi_{\text{odd}}(G) \leq q$ is

- polynomial-time solvable if $q \leq 2$, and
- NP-complete if $q \geq 3$.

Since computing $\text{mos}(G)$ and $\chi_{\text{odd}}(G)$ are NP-hard, we focus on its parameterized complexity,

For an integer $q \geq 1$, we prove that deciding whether $\chi_{\mathsf{odd}}(G) \leq q$ is

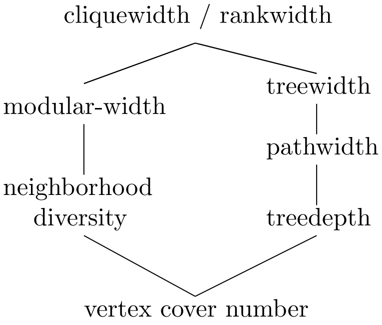
- polynomial-time solvable if $q \leq 2$, and
- NP-complete if $q \geq 3$.

Since computing $\mathsf{mos}(G)$ and $\chi_{\mathsf{odd}}(G)$ are NP-hard, we focus on its parameterized complexity, in particular on structural parameters.

For an integer $q \geq 1$, we prove that deciding whether $\chi_{\mathsf{odd}}(G) \leq q$ is

- polynomial-time solvable if $q \leq 2$, and
- NP-complete if $q \geq 3$.

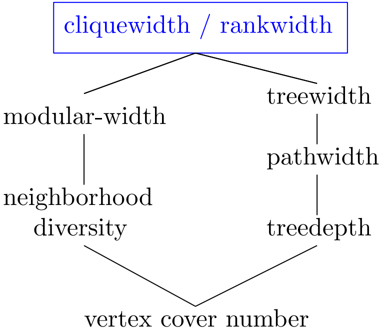
Since computing $\mathsf{mos}(G)$ and $\chi_{\mathsf{odd}}(G)$ are NP-hard, we focus on its parameterized complexity, in particular on structural parameters.



For an integer $q \geq 1$, we prove that deciding whether $\chi_{\mathsf{odd}}(G) \leq q$ is

- polynomial-time solvable if $q \leq 2$, and
- NP-complete if $q \geq 3$.

Since computing $\mathsf{mos}(G)$ and $\chi_{\mathsf{odd}}(G)$ are NP-hard, we focus on its parameterized complexity, in particular on structural parameters.



We present FPT algorithms parameterized by rankwidth, in time:

- $2^{\mathcal{O}(\mathsf{rw})} \cdot n^{\mathcal{O}(1)}$ for computing $\mathsf{mes}(G)$ and $\mathsf{mos}(G)$,
- $2^{\mathcal{O}(q \cdot \mathsf{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathsf{odd}}(G) \leq q$,

for an n-vertex graph G, given a decomposition tree of width at most rw.

We present FPT algorithms parameterized by rankwidth, in time:

- $2^{\mathcal{O}(\mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for computing $\mathrm{mes}(G)$ and $\mathrm{mos}(G)$,
- $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$,

for an n-vertex graph G, given a decomposition tree of width at most rw.

- Inspired by algorithms of [Bui-Xuan, Telle, Vatshelle. 2010-2011-2013]

We present FPT algorithms parameterized by rankwidth, in time:

- $2^{\mathcal{O}(\mathsf{rw})} \cdot n^{\mathcal{O}(1)}$ for computing $\mathsf{mes}(G)$ and $\mathsf{mos}(G)$,
- $2^{\mathcal{O}(q \cdot \mathsf{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathsf{odd}}(G) \leq q$,

for an n-vertex graph G, given a decomposition tree of width at most rw.

- Inspired by algorithms of [Bui-Xuan, Telle, Vatshelle. 2010-2011-2013]
- First algorithms in time $2^{o(\mathsf{rw}^2)} \cdot n^{\mathcal{O}(1)}$ for an NP-hard problem.

We present FPT algorithms parameterized by rankwidth, in time:

- $2^{\mathcal{O}(\mathsf{rw})} \cdot n^{\mathcal{O}(1)}$ for computing $\mathsf{mes}(G)$ and $\mathsf{mos}(G)$,
- $2^{\mathcal{O}(q \cdot \mathsf{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathsf{odd}}(G) \leq q$,

for an n-vertex graph G, given a decomposition tree of width at most rw.

- Inspired by algorithms of [Bui-Xuan, Telle, Vatshelle. 2010-2011-2013]
- First algorithms in time $2^{o(\mathsf{rw}^2)} \cdot n^{\mathcal{O}(1)}$ for an NP-hard problem.
- Is the function $2^{\mathcal{O}(\mathsf{rw})}$ optimal under the ETH? ($\nexists 2^{o(n)}$ algo for 3-SAT)

We present FPT algorithms parameterized by rankwidth, in time:

- $2^{\mathcal{O}(\mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for computing $\mathrm{mes}(G)$ and $\mathrm{mos}(G)$,
- $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$,

for an n-vertex graph G, given a decomposition tree of width at most rw.

- Inspired by algorithms of [Bui-Xuan, Telle, Vatshelle. 2010-2011-2013]
- First algorithms in time $2^{o(\mathrm{rw}^2)} \cdot n^{\mathcal{O}(1)}$ for an NP-hard problem.
- Is the function $2^{\mathcal{O}(\mathrm{rw})}$ optimal under the ETH? ($\nexists 2^{o(n)}$ algo for 3-SAT)
  - For deciding whether $\chi_{\mathrm{odd}}(G) \leq q$, our NP-hardness reduction implies that $\nexists$ $2^{o(n)}$ algo under the ETH

We present FPT algorithms parameterized by rankwidth, in time:

- $2^{\mathcal{O}(\mathsf{rw})} \cdot n^{\mathcal{O}(1)}$ for computing $\mathsf{mes}(G)$ and $\mathsf{mos}(G)$,
- $2^{\mathcal{O}(q \cdot \mathsf{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathsf{odd}}(G) \leq q$,

for an n-vertex graph G, given a decomposition tree of width at most rw.

- Inspired by algorithms of [Bui-Xuan, Telle, Vatshelle. 2010-2011-2013]
- First algorithms in time $2^{o(\mathsf{rw}^2)} \cdot n^{\mathcal{O}(1)}$ for an NP-hard problem.
- Is the function $2^{\mathcal{O}(\mathsf{rw})}$ optimal under the ETH? ($\nexists 2^{o(n)}$ algo for 3-SAT)
    - For deciding whether $\chi_{\mathsf{odd}}(G) \leq q$, our NP-hardness reduction implies that $\nexists\ 2^{o(n)}$ algo under the ETH $\Rightarrow$ $\nexists\ 2^{o(\mathsf{rw})} \cdot n^{\mathcal{O}(1)}$ ✓

We present FPT algorithms parameterized by rankwidth, in time:

- $2^{\mathcal{O}(\mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for computing $\mathrm{mes}(G)$ and $\mathrm{mos}(G)$,
- $2^{\mathcal{O}(q\cdot\mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$,

for an n-vertex graph G, given a decomposition tree of width at most rw.

- Inspired by algorithms of [Bui-Xuan, Telle, Vatshelle. 2010-2011-2013]
- First algorithms in time $2^{o(\mathrm{rw}^2)} \cdot n^{\mathcal{O}(1)}$ for an NP-hard problem.
- Is the function $2^{\mathcal{O}(\mathrm{rw})}$ optimal under the ETH? ($\nexists 2^{o(n)}$ algo for 3-SAT)
  - For deciding whether $\chi_{\mathrm{odd}}(G) \leq q$, our NP-hardness reduction implies that $\nexists\ 2^{o(n)}$ algo under the ETH $\Rightarrow$ $\nexists\ 2^{o(\mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ ✓
  - For computing $\mathrm{mes}(G)$ and $\mathrm{mos}(G)$, existing NP-hardness reduction implies only that $\nexists\ 2^{o(\sqrt{n})}$ algo under the ETH. [Cai, Yang. 2011]

We present FPT algorithms parameterized by rankwidth, in time:

- $2^{\mathcal{O}(\mathsf{rw})} \cdot n^{\mathcal{O}(1)}$ for computing $\mathsf{mes}(G)$ and $\mathsf{mos}(G)$,
- $2^{\mathcal{O}(q \cdot \mathsf{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathsf{odd}}(G) \leq q$,

for an n-vertex graph G, given a decomposition tree of width at most rw.

- Inspired by algorithms of [Bui-Xuan, Telle, Vatshelle. 2010-2011-2013]
- First algorithms in time $2^{o(\mathsf{rw}^2)} \cdot n^{\mathcal{O}(1)}$ for an NP-hard problem.
- Is the function $2^{\mathcal{O}(\mathsf{rw})}$ optimal under the ETH? ($\nexists 2^{o(n)}$ algo for 3-SAT)
  - For deciding whether $\chi_{\mathsf{odd}}(G) \leq q$, our NP-hardness reduction implies that $\nexists\ 2^{o(n)}$ algo under the ETH $\Rightarrow$ $\nexists\ 2^{o(\mathsf{rw})} \cdot n^{\mathcal{O}(1)}$ ✓
  - For computing $\mathsf{mes}(G)$ and $\mathsf{mos}(G)$, existing NP-hardness reduction implies only that $\nexists\ 2^{o(\sqrt{n})}$ algo under the ETH. [Cai, Yang. 2011]

    We provide a linear NP-hardness reduction for $\mathsf{mes}(G)$ and $\mathsf{mos}(G)$,

We present FPT algorithms parameterized by rankwidth, in time:

- $2^{\mathcal{O}(\mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for computing $\mathrm{mes}(G)$ and $\mathrm{mos}(G)$,
- $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$,

for an n-vertex graph G, given a decomposition tree of width at most rw.

- Inspired by algorithms of [Bui-Xuan, Telle, Vatshelle. 2010-2011-2013]
- First algorithms in time $2^{o(\mathrm{rw}^2)} \cdot n^{\mathcal{O}(1)}$ for an NP-hard problem.
- Is the function $2^{\mathcal{O}(\mathrm{rw})}$ optimal under the ETH? ($\nexists 2^{o(n)}$ algo for 3-SAT)
  - For deciding whether $\chi_{\mathrm{odd}}(G) \leq q$, our NP-hardness reduction implies that $\nexists\ 2^{o(n)}$ algo under the ETH $\Rightarrow$ $\nexists\ 2^{o(\mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ ✓
  - For computing $\mathrm{mes}(G)$ and $\mathrm{mos}(G)$, existing NP-hardness reduction implies only that $\nexists\ 2^{o(\sqrt{n})}$ algo under the ETH. [Cai, Yang. 2011]

    We provide a linear NP-hardness reduction for $\mathrm{mes}(G)$ and $\mathrm{mos}(G)$, hence $\nexists\ 2^{o(n)}$ algo under ETH

We present FPT algorithms parameterized by rankwidth, in time:

- $2^{\mathcal{O}(\mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for computing $\mathrm{mes}(G)$ and $\mathrm{mos}(G)$,
- $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$,

for an n-vertex graph G, given a decomposition tree of width at most rw.

- Inspired by algorithms of [Bui-Xuan, Telle, Vatshelle. 2010-2011-2013]
- First algorithms in time $2^{o(\mathrm{rw}^2)} \cdot n^{\mathcal{O}(1)}$ for an NP-hard problem.
- Is the function $2^{\mathcal{O}(\mathrm{rw})}$ optimal under the ETH? ($\nexists 2^{o(n)}$ algo for 3-SAT)
  - For deciding whether $\chi_{\mathrm{odd}}(G) \leq q$, our NP-hardness reduction implies that $\nexists\ 2^{o(n)}$ algo under the ETH $\Rightarrow$ $\nexists\ 2^{o(\mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ ✓
  - For computing $\mathrm{mes}(G)$ and $\mathrm{mos}(G)$, existing NP-hardness reduction implies only that $\nexists\ 2^{o(\sqrt{n})}$ algo under the ETH. [Cai, Yang. 2011]

    We provide a linear NP-hardness reduction for $\mathrm{mes}(G)$ and $\mathrm{mos}(G)$, hence $\nexists\ 2^{o(n)}$ algo under ETH $\Rightarrow$ $\nexists\ 2^{o(\mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ ✓

Finally, we provide bounds on the parameters $\chi_{\mathrm{odd}}(G)$ and $\mathrm{mos}(G)$.

Finally, we provide bounds on the parameters $\chi_{\text{odd}}(G)$ and $\text{mos}(G)$.

1. We prove that, for every graph G with all components of even order,

$$\chi_{\text{odd}}(G) \leq \text{tw}(G) + 1.$$

Finally, we provide bounds on the parameters $\chi_{\mathrm{odd}}(G)$ and $\mathrm{mos}(G)$.

1. We prove that, for every graph G with all components of even order,

$$\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1.$$

   This bound is tight and has some consequences.

Finally, we provide bounds on the parameters $\chi_{\text{odd}}(G)$ and $\text{mos}(G)$.

1. We prove that, for every graph G with all components of even order,

$$\chi_{\text{odd}}(G) \leq \text{tw}(G) + 1.$$

   This bound is tight and has some consequences.

   Recall the "folklore" conjecture about $\text{mos}(G)$:

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\text{mos}(G) \geq c \cdot n$.

Finally, we provide bounds on the parameters $\chi_{\text{odd}}(G)$ and $\text{mos}(G)$.

1. We prove that, for every graph G with all components of even order,

$$\chi_{\text{odd}}(G) \leq \text{tw}(G) + 1.$$

   This bound is tight and has some consequences.

   Recall the "folklore" conjecture about $\text{mos}(G)$:

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\text{mos}(G) \geq c \cdot n$.

- $\mathcal{G}_k$: graphs of treewidth at most k without isolated vertices.

Finally, we provide bounds on the parameters $\chi_{\mathrm{odd}}(G)$ and $\mathrm{mos}(G)$.

1. We prove that, for every graph G with all components of even order,

$$\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1.$$

   This bound is tight and has some consequences.

   Recall the "folklore" conjecture about $\mathrm{mos}(G)$:

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\mathrm{mos}(G) \geq c \cdot n$.

- $\mathcal{G}_k$: graphs of treewidth at most k without isolated vertices.
- $c_k = \min_{G \in \mathcal{G}_k} \frac{\mathrm{mos}(G)}{|V(G)|}$.

Finally, we provide bounds on the parameters $\chi_{\text{odd}}(G)$ and $\text{mos}(G)$.

1. We prove that, for every graph G with all components of even order,

$$\chi_{\text{odd}}(G) \leq \text{tw}(G) + 1.$$

This bound is tight and has some consequences.

Recall the "folklore" conjecture about $\text{mos}(G)$:

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\text{mos}(G) \geq c \cdot n$.

- $\mathcal{G}_k$: graphs of treewidth at most k without isolated vertices.
- $c_k = \min_{G \in \mathcal{G}_k} \frac{\text{mos}(G)}{|V(G)|}$.
- So, $c_k > 0$ if and only if the conjecture is true for $\mathcal{G}_k$.

Finally, we provide bounds on the parameters $\chi_{\text{odd}}(G)$ and $\text{mos}(G)$.

1. We prove that, for every graph G with all components of even order,

$$\chi_{\text{odd}}(G) \leq \text{tw}(G) + 1.$$

This bound is tight and has some consequences.

Recall the "folklore" conjecture about $\text{mos}(G)$:

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\text{mos}(G) \geq c \cdot n$.

- $\mathcal{G}_k$: graphs of treewidth at most k without isolated vertices.
- $c_k = \min_{G \in \mathcal{G}_k} \frac{\text{mos}(G)}{|V(G)|}$.
- So, $c_k > 0$ if and only if the conjecture is true for $\mathcal{G}_k$.
- It is known that $c_k \geq \frac{1}{2(k+1)}$. [Scott. 1992]

Finally, we provide bounds on the parameters $\chi_{\text{odd}}(G)$ and $\text{mos}(G)$.

1. We prove that, for every graph G with all components of even order,

$$\chi_{\text{odd}}(G) \leq \text{tw}(G) + 1.$$

This bound is tight and has some consequences.

Recall the "folklore" conjecture about $\text{mos}(G)$:

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\text{mos}(G) \geq c \cdot n$.

- $\mathcal{G}_k$: graphs of treewidth at most k without isolated vertices.
- $c_k = \min_{G \in \mathcal{G}_k} \frac{\text{mos}(G)}{|V(G)|}$.
- So, $c_k > 0$ if and only if the conjecture is true for $\mathcal{G}_k$.
- It is known that $c_k \geq \frac{1}{2(k+1)}$. [Scott. 1992]
- Our bound implies that $c_k \geq \frac{1}{k+1}$.

Conjecture

There exists a constant $c > 0$ *such that, for every* n*-vertex graph* G *without isolated vertices,* $\mathrm{mos}(G) \geq c \cdot n$.

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\mathrm{mos}(G) \geq c \cdot n$.

Proved for particular graph classes, with best possible constant $c > 0$:

- Trees. [Radcliffe, Scott. 1995]
- Graphs G with bounded $\chi(G)$. [Scott. 1992]
- Graphs G with $\Delta(G) \leq 3$. [Berman, Wang, Wargo. 1997]
- Graphs G with $\mathrm{tw}(G) \leq 2$. [Hou, Yu, Li, Liu. 2018]

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\mathrm{mos}(G) \geq c \cdot n$.

Proved for particular graph classes, with best possible constant $c > 0$:

- Trees. [Radcliffe, Scott. 1995]
- Graphs G with bounded $\chi(G)$. [Scott. 1992]
- Graphs G with $\Delta(G) \leq 3$. [Berman, Wang, Wargo. 1997]
- Graphs G with $\mathrm{tw}(G) \leq 2$. [Hou, Yu, Li, Liu. 2018]

- We prove that if $\mathrm{cw}(G) \leq 2$ (cographs), then

$$\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil,$$

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\mathrm{mos}(G) \geq c \cdot n$.

Proved for particular graph classes, with best possible constant $c > 0$:

- Trees. [Radcliffe, Scott. 1995]
- Graphs G with bounded $\chi(G)$. [Scott. 1992]
- Graphs G with $\Delta(G) \leq 3$. [Berman, Wang, Wargo. 1997]
- Graphs G with $\mathrm{tw}(G) \leq 2$. [Hou, Yu, Li, Liu. 2018]

- We prove that if $\mathrm{cw}(G) \leq 2$ (cographs), then

$$\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil, \quad \text{and this bound is tight.}$$

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\mathrm{mos}(G) \geq c \cdot n$.

Proved for particular graph classes, with best possible constant $c > 0$:

- Trees. [Radcliffe, Scott. 1995]
- Graphs G with bounded $\chi(G)$. [Scott. 1992]
- Graphs G with $\Delta(G) \leq 3$. [Berman, Wang, Wargo. 1997]
- Graphs G with $\mathrm{tw}(G) \leq 2$. [Hou, Yu, Li, Liu. 2018]

- We prove that if $\mathrm{cw}(G) \leq 2$ (cographs), then

$$\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil,$$ and this bound is tight.

- We prove that, if G is a cograph, then $\chi_{\mathrm{odd}}(G) \leq 3$, and this is tight.

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\mathrm{mos}(G) \geq c \cdot n$.

Proved for particular graph classes, with best possible constant $c > 0$:

- Trees. [Radcliffe, Scott. 1995]
- Graphs G with bounded $\chi(G)$. [Scott. 1992]
- Graphs G with $\Delta(G) \leq 3$. [Berman, Wang, Wargo. 1997]
- Graphs G with $\mathrm{tw}(G) \leq 2$. [Hou, Yu, Li, Liu. 2018]

- We prove that if $\mathrm{cw}(G) \leq 2$ (cographs), then

$$\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil, \quad \text{and this bound is tight.}$$

- We prove that, if G is a cograph, then $\chi_{\mathrm{odd}}(G) \leq 3$, and this is tight.

- Note that cographs are exactly P_4-free graphs.

Conjecture

There exists a constant $c > 0$ such that, for every n-vertex graph G without isolated vertices, $\text{mos}(G) \geq c \cdot n$.

Proved for particular graph classes, with best possible constant $c > 0$:

- Trees. [Radcliffe, Scott. 1995]
- Graphs G with bounded $\chi(G)$. [Scott. 1992]
- Graphs G with $\Delta(G) \leq 3$. [Berman, Wang, Wargo. 1997]
- Graphs G with $\text{tw}(G) \leq 2$. [Hou, Yu, Li, Liu. 2018]

- We prove that if $\text{cw}(G) \leq 2$ (cographs), then

$$\text{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil, \quad \text{and this bound is tight.}$$

- We prove that, if G is a cograph, then $\chi_{\text{odd}}(G) \leq 3$, and this is tight.

- Note that cographs are exactly P_4-free graphs.
  We show that χ_{odd} is unbounded for P_5-free graphs.

skip

Next section is...

Theorem

For an integer $q \geq 1$, deciding whether $\chi_{\mathrm{odd}}(G) \leq q$ is

- *polynomial-time solvable if $q \leq 2$, and*
- NP-*complete if $q \geq 3$.*

Theorem

For an integer $q \geq 1$, deciding whether $\chi_{\text{odd}}(G) \leq q$ is

- *polynomial-time solvable if $q \leq 2$, and*
- *NP-complete if $q \geq 3$.*

For $q = 1$ the problem is trivial: G needs to be an odd graph itself.

Theorem

For an integer $q \geq 1$, deciding whether $\chi_{\text{odd}}(G) \leq q$ is

- *polynomial-time solvable if $q \leq 2$, and*
- NP-*complete if $q \geq 3$.*

For $q = 2 \equiv$ feasibility of a system of linear equations over GF[2]:

Theorem

For an integer $q \geq 1$, deciding whether $\chi_{\text{odd}}(G) \leq q$ is

- *polynomial-time solvable if $q \leq 2$, and*
- *NP-complete if $q \geq 3$.*

For $q = 2 \equiv$ feasibility of a system of linear equations over GF[2]:

- $V(G) = \{v_1, \ldots, v_n\}$.

Theorem

For an integer $q \geq 1$, deciding whether $\chi_{\mathrm{odd}}(G) \leq q$ is

- *polynomial-time solvable if $q \leq 2$, and*
- NP-*complete if $q \geq 3$.*

For $q = 2 \equiv$ feasibility of a system of linear equations over GF[2]:

- $V(G) = \{v_1, \ldots, v_n\}$. Want $V(G) = V_0 \uplus V_1$ with $G[V_0], G[V_1]$ odd.

Theorem

For an integer $q \geq 1$, deciding whether $\chi_{\text{odd}}(G) \leq q$ is

- *polynomial-time solvable if $q \leq 2$, and*
- *NP-complete if $q \geq 3$.*

For $q = 2 \equiv$ feasibility of a system of linear equations over GF[2]:

- $V(G) = \{v_1, \ldots, v_n\}$. Want $V(G) = V_0 \uplus V_1$ with $G[V_0]$,$G[V_1]$ odd.
- For every vertex v_i, create a binary variable x_i.
  For every edge $v_i v_j$, create a binary variable $x_{i,j}$.

Theorem

For an integer $q \geq 1$, deciding whether $\chi_{\text{odd}}(G) \leq q$ is

- *polynomial-time solvable if $q \leq 2$, and*
- *NP-complete if $q \geq 3$.*

For $q = 2 \equiv$ feasibility of a system of linear equations over GF[2]:

- $V(G) = \{v_1, \ldots, v_n\}$. Want $V(G) = V_0 \uplus V_1$ with $G[V_0]$,$G[V_1]$ odd.
- For every vertex v_i, create a binary variable x_i.
  For every edge $v_i v_j$, create a binary variable $x_{i,j}$.
- x_i: indicates whether $v_i \in V_0$ or $v_i \in V_1$.

Theorem

For an integer $q \geq 1$, deciding whether $\chi_{\text{odd}}(G) \leq q$ is

- *polynomial-time solvable if $q \leq 2$, and*
- NP-*complete if $q \geq 3$.*

For $q = 2 \equiv$ feasibility of a system of linear equations over GF[2]:

- $V(G) = \{v_1, \ldots, v_n\}$. Want $V(G) = V_0 \uplus V_1$ with $G[V_0]$,$G[V_1]$ odd.
- For every vertex v_i, create a binary variable x_i.
  For every edge $v_i v_j$, create a binary variable $x_{i,j}$.
- x_i: indicates whether $v_i \in V_0$ or $v_i \in V_1$.
  $x_{i,j}$: indicates whether $v_i v_j$ is monochromatic (1) or not (0).

Theorem

For an integer $q \geq 1$, deciding whether $\chi_{\text{odd}}(G) \leq q$ is

- *polynomial-time solvable if $q \leq 2$, and*
- *NP-complete if $q \geq 3$.*

For $q = 2 \equiv$ feasibility of a system of linear equations over GF[2]:

- $V(G) = \{v_1, \ldots, v_n\}$. Want $V(G) = V_0 \uplus V_1$ with $G[V_0], G[V_1]$ odd.
- For every vertex v_i, create a binary variable x_i.
  For every edge $v_i v_j$, create a binary variable $x_{i,j}$.
- x_i: indicates whether $v_i \in V_0$ or $v_i \in V_1$.
  $x_{i,j}$: indicates whether $v_i v_j$ is monochromatic (1) or not (0).

$$\begin{cases} x_i + x_j + x_{i,j} \equiv 1 & \text{for every edge } v_i v_j \in E(G) \\ & \end{cases}$$

Theorem

For an integer $q \geq 1$, deciding whether $\chi_{\text{odd}}(G) \leq q$ is

- *polynomial-time solvable if $q \leq 2$, and*
- *NP-complete if $q \geq 3$.*

For $q = 2 \equiv$ feasibility of a system of linear equations over GF[2]:

- $V(G) = \{v_1, \ldots, v_n\}$. Want $V(G) = V_0 \uplus V_1$ with $G[V_0], G[V_1]$ odd.
- For every vertex v_i, create a binary variable x_i.
  For every edge $v_i v_j$, create a binary variable $x_{i,j}$.
- x_i: indicates whether $v_i \in V_0$ or $v_i \in V_1$.
  $x_{i,j}$: indicates whether $v_i v_j$ is monochromatic (1) or not (0).

$$\begin{cases} x_i + x_j + x_{i,j} \equiv 1 & \text{for every edge } v_i v_j \in E(G) \\ \sum_{j : v_j \in N(v_i)} x_{i,j} \equiv 1 & \text{for every vertex } v_i \in V(G) \end{cases}$$

Theorem

For an integer $q \geq 1$, deciding whether $\chi_{\text{odd}}(G) \leq q$ is

- *polynomial-time solvable if $q \leq 2$, and*
- *NP-complete if $q \geq 3$.*

For $q = 2 \equiv$ feasibility of a system of linear equations over GF[2]:

- $V(G) = \{v_1, \ldots, v_n\}$. Want $V(G) = V_0 \uplus V_1$ with $G[V_0], G[V_1]$ odd.
- For every vertex v_i, create a binary variable x_i.
  For every edge $v_i v_j$, create a binary variable $x_{i,j}$.
- x_i: indicates whether $v_i \in V_0$ or $v_i \in V_1$.
  $x_{i,j}$: indicates whether $v_i v_j$ is monochromatic (1) or not (0).

$$\begin{cases} x_i + x_j + x_{i,j} \equiv 1 & \text{for every edge } v_i v_j \in E(G) \\ \sum_{j : v_j \in N(v_i)} x_{i,j} \equiv 1 & \text{for every vertex } v_i \in V(G) \end{cases}$$

- $\chi_{\text{odd}}(G) \leq 2 \iff$ the above system is feasible.

skip

$\boxed{\text{Deciding whether } \chi_{\mathrm{odd}}(G) \leq q \text{ is NP-complete if } q \geq 3.}$

$\boxed{\text{Deciding whether } \chi_{\text{odd}}(G) \leq q \text{ is NP-complete if } q \geq 3.}$

We reduce from q-Coloring.

$\boxed{\text{Deciding whether } \chi_{\mathsf{odd}}(G) \leq q \text{ is NP-complete if } q \geq 3.}$

We reduce from q-Coloring. Suppose $q = 3$.

$\boxed{\text{Deciding whether } \chi_{\mathsf{odd}}(G) \leq q \text{ is NP-complete if } q \geq 3.}$

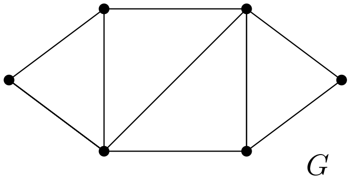
We reduce from q-COLORING. Suppose $q = 3$.

★ Any graph $G = (V, E)$ such that $|V| + |E|$ is even admits an orientation of E such that all vertex in-degrees are odd. [Frank, Jordán, Szigeti. 1999]

$\boxed{\text{Deciding whether } \chi_{\text{odd}}(G) \leq q \text{ is NP-complete if } q \geq 3.}$

We reduce from q-Coloring. Suppose $q = 3$.

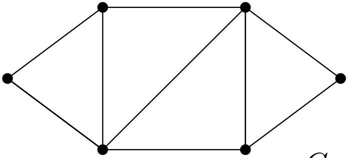
★ Any graph $G = (V, E)$ such that $|V| + |E|$ is even admits an orientation of E such that all vertex in-degrees are odd. [Frank, Jordán, Szigeti. 1999]



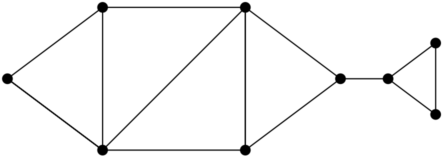
Deciding whether $\chi_{\text{odd}}(G) \leq q$ is NP-complete if $q \geq 3$.

We reduce from q-Coloring. Suppose $q = 3$.

★ Any graph $G = (V, E)$ such that $|V| + |E|$ is even admits an orientation of E such that all vertex in-degrees are odd. [Frank, Jordán, Szigeti. 1999]



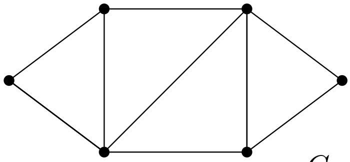
G



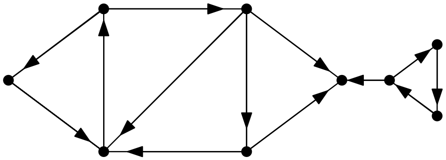
$\boxed{\text{Deciding whether } \chi_{\mathsf{odd}}(G) \leq q \text{ is NP-complete if } q \geq 3.}$

We reduce from q-Coloring. Suppose $q = 3$.

★ Any graph $G = (V, E)$ such that $|V| + |E|$ is even admits an orientation of E such that all vertex in-degrees are odd. [Frank, Jordán, Szigeti. 1999]



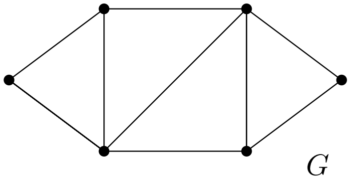
G

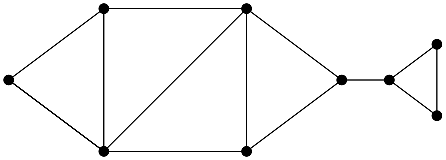


Deciding whether $\chi_{\mathsf{odd}}(G) \leq q$ is NP-complete if $q \geq 3$.

We reduce from q-Coloring. Suppose $q = 3$.

★ Any graph $G = (V, E)$ such that $|V| + |E|$ is even admits an orientation of E such that all vertex in-degrees are odd. [Frank, Jordán, Szigeti. 1999]

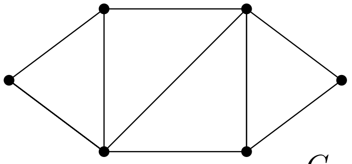




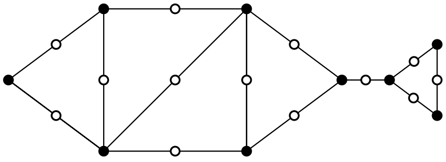
$\boxed{\text{Deciding whether } \chi_{\text{odd}}(G) \le q \text{ is NP-complete if } q \ge 3.}$

We reduce from q-Coloring. Suppose $q = 3$.

★ Any graph $G = (V, E)$ such that $|V| + |E|$ is even admits an orientation of E such that all vertex in-degrees are odd. [Frank, Jordán, Szigeti. 1999]



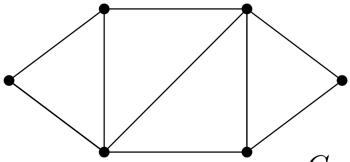
G



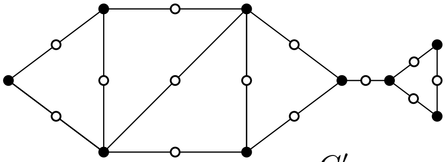
Deciding whether $\chi_{\mathrm{odd}}(G) \leq q$ is NP-complete if $q \geq 3$.

We reduce from q-Coloring. Suppose $q = 3$.

★ Any graph $G = (V, E)$ such that $|V| + |E|$ is even admits an orientation of E such that all vertex in-degrees are odd. [Frank, Jordán, Szigeti. 1999]



G

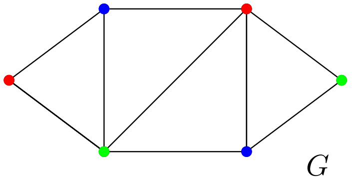


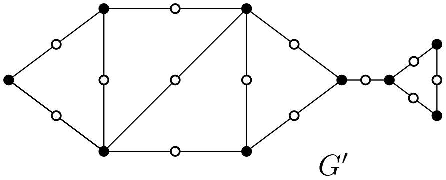
G'

$\boxed{\text{Deciding whether } \chi_{\mathrm{odd}}(G) \leq q \text{ is NP-complete if } q \geq 3.}$

We reduce from q-Coloring. Suppose $q = 3$.

★ Any graph $G = (V, E)$ such that $|V| + |E|$ is even admits an orientation of E such that all vertex in-degrees are odd. [Frank, Jordán, Szigeti. 1999]

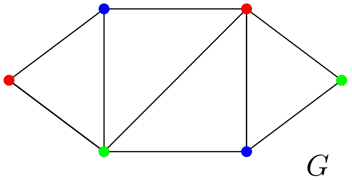


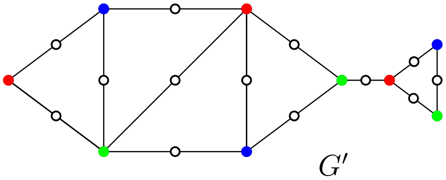


$\boxed{\text{Deciding whether } \chi_{\text{odd}}(G) \leq q \text{ is NP-complete if } q \geq 3.}$

We reduce from q-Coloring. Suppose $q = 3$.

★ Any graph $G = (V, E)$ such that $|V| + |E|$ is even admits an orientation of E such that all vertex in-degrees are odd. [Frank, Jordán, Szigeti. 1999]

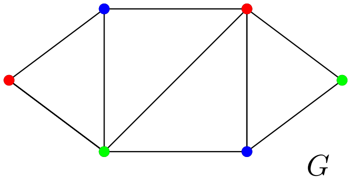


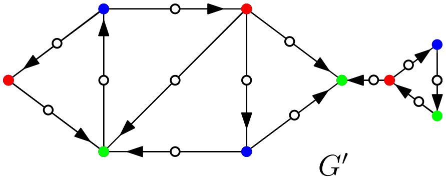


$\boxed{\text{Deciding whether } \chi_{\text{odd}}(G) \leq q \text{ is NP-complete if } q \geq 3.}$

We reduce from q-COLORING. Suppose $q = 3$.

★ Any graph $G = (V, E)$ such that $|V| + |E|$ is even admits an orientation of E such that all vertex in-degrees are odd. [Frank, Jordán, Szigeti. 1999]

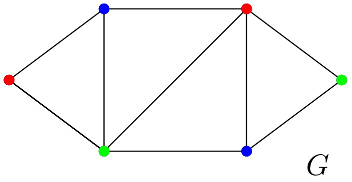


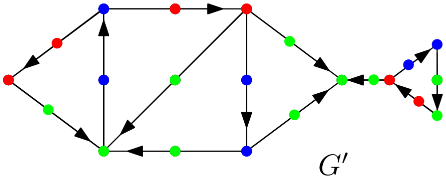


$\boxed{\text{Deciding whether } \chi_{\mathsf{odd}}(G) \leq q \text{ is NP-complete if } q \geq 3.}$

We reduce from q-Coloring. Suppose $q = 3$.

★ Any graph $G = (V, E)$ such that $|V| + |E|$ is even admits an orientation of E such that all vertex in-degrees are odd. [Frank, Jordán, Szigeti. 1999]

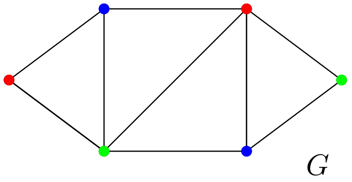


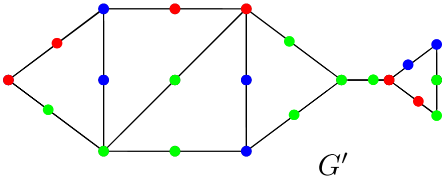


Deciding whether $\chi_{\text{odd}}(G) \leq q$ is NP-complete if $q \geq 3$.

We reduce from q-COLORING. Suppose $q = 3$.

★ Any graph $G = (V, E)$ such that $|V| + |E|$ is even admits an orientation of E such that all vertex in-degrees are odd. [Frank, Jordán, Szigeti. 1999]

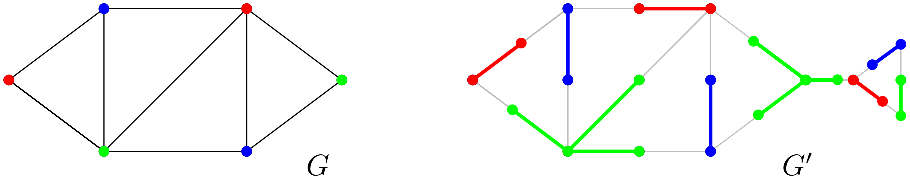




$\boxed{\text{Deciding whether } \chi_{\mathrm{odd}}(G) \leq q \text{ is NP-complete if } q \geq 3.}$

We reduce from q-COLORING. Suppose $q = 3$.

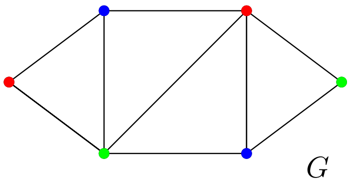
★ Any graph $G = (V, E)$ such that $|V| + |E|$ is even admits an orientation of E such that all vertex in-degrees are odd. [Frank, Jordán, Szigeti. 1999]

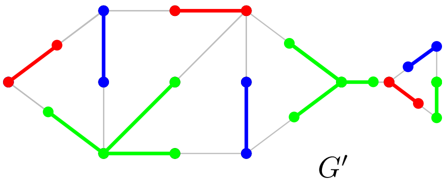


$\boxed{\text{Deciding whether } \chi_{\text{odd}}(G) \le q \text{ is NP-complete if } q \ge 3.}$

We reduce from q-COLORING. Suppose $q = 3$.

★ Any graph $G = (V, E)$ such that $|V| + |E|$ is even admits an orientation of E such that all vertex in-degrees are odd. [Frank, Jordán, Szigeti. 1999]





Thus, G is 3-colorable $\iff$ $\chi_{\text{odd}}(G') \le 3$.

skip □

Theorem

For every graph G with all components of even order we have that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1$, and this bound is tight.

Theorem

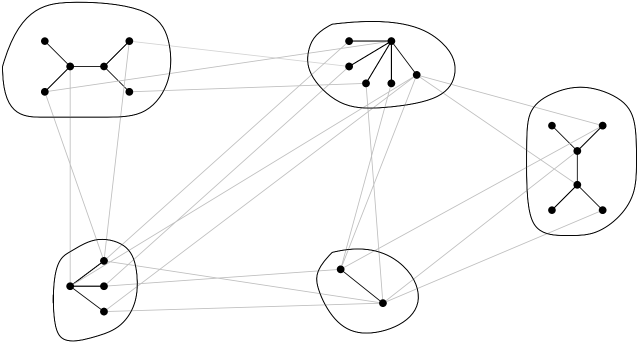
For every graph G with all components of even order we have that $\chi_{\text{odd}}(G) \leq \text{tw}(G) + 1$, and this bound is tight.

★ Every graph G with all components of even order admits a vertex partition such that every vertex class induces an odd tree. [Scott. 2001]

Theorem

For every graph G with all components of even order we have that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1$, and this bound is tight.

★ Every graph G with all components of even order admits a vertex partition such that every vertex class induces an odd tree. [Scott. 2001]

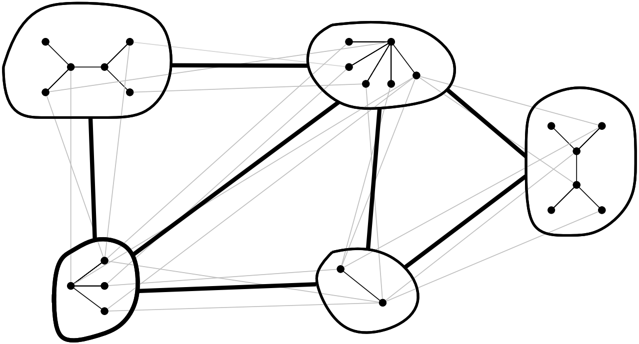


Given G, consider a partition of $V(G)$ into induced odd trees.

Theorem

For every graph G with all components of even order we have that $\chi_{\text{odd}}(G) \leq \text{tw}(G) + 1$, and this bound is tight.

★ Every graph G with all components of even order admits a vertex partition such that every vertex class induces an odd tree. [Scott. 2001]

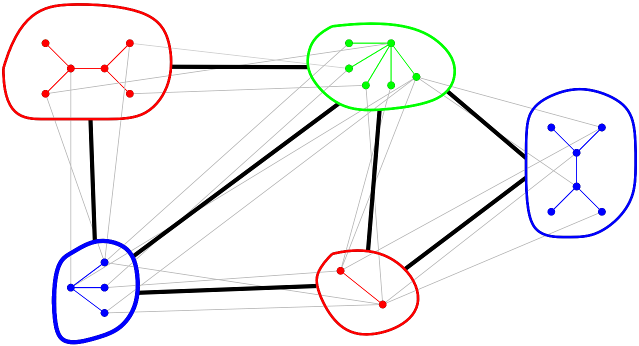


Let G' be obtained from G by contracting each tree to a single vertex.

Theorem

For every graph G with all components of even order we have that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G)+1$, and this bound is tight.

★ Every graph G with all components of even order admits a vertex partition such that every vertex class induces an odd tree. [Scott. 2001]

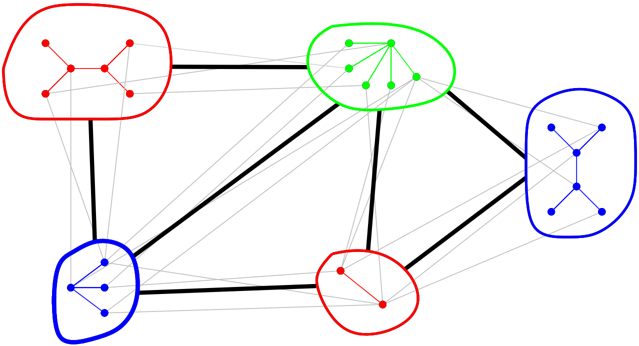


Consider a proper vertex coloring of G' using $\chi(G')$ colors.

Theorem

For every graph G with all components of even order we have that $\chi_{\text{odd}}(G) \leq \text{tw}(G) + 1$, and this bound is tight.

★ Every graph G with all components of even order admits a vertex partition such that every vertex class induces an odd tree. [Scott. 2001]

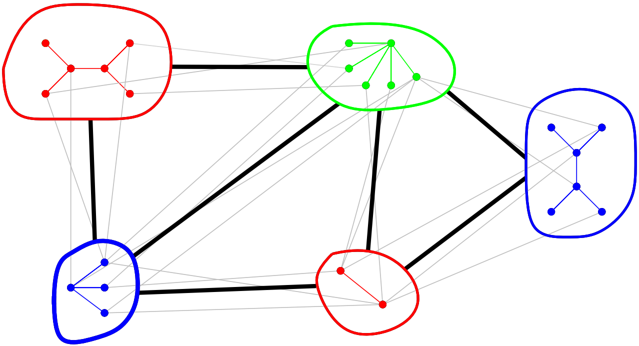


We have that $\chi_{\text{odd}}(G) \leq \chi(G')$

Theorem

For every graph G with all components of even order we have that $\chi_{\text{odd}}(G) \leq \text{tw}(G) + 1$, and this bound is tight.

★ Every graph G with all components of even order admits a vertex partition such that every vertex class induces an odd tree. [Scott. 2001]

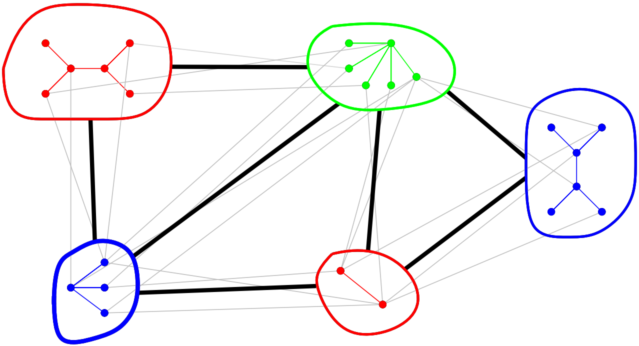


We have that $\chi_{\text{odd}}(G) \leq \chi(G') \leq \text{tw}(G') + 1$

Theorem

For every graph G with all components of even order we have that $\chi_{\text{odd}}(G) \leq \text{tw}(G) + 1$, and this bound is tight.

★ Every graph G with all components of even order admits a vertex partition such that every vertex class induces an odd tree. [Scott. 2001]

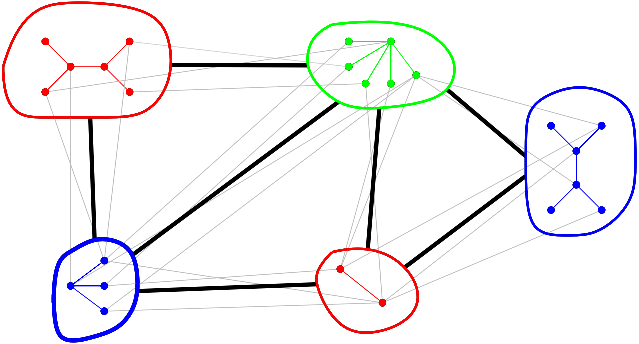


We have that $\chi_{\text{odd}}(G) \leq \chi(G') \leq \text{tw}(G') + 1 \leq \text{tw}(G) + 1$. $\square$

Theorem

For every graph G with all components of even order we have that $\chi_{\text{odd}}(G) \leq \text{tw}(G) + 1$, and this bound is tight.

★ Every graph G with all components of even order admits a vertex partition such that every vertex class induces an odd tree. [Scott. 2001]



Bound is tight: let G be subdivided n-clique with $n \equiv 0, 3 \pmod 4$.

skip

Theorem

If $\mathrm{cw}(G) \leq 2$ *(cograph), then* $\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil$*, and this bound is tight.*

Theorem

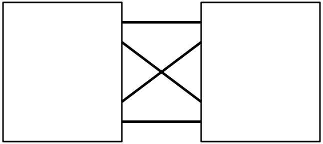
If $\mathrm{cw}(G) \leq 2$ *(cograph), then* $\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil$*, and this bound is tight.*

Every n-vertex graph G that admits a join satisfies $\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil$.

Theorem

If $\mathrm{cw}(G) \leq 2$ *(cograph), then* $\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil$*, and this bound is tight.*

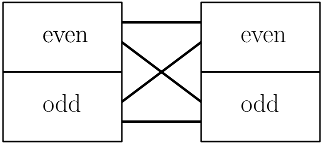
Every n-vertex graph G that admits a join satisfies $\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil$.



Theorem

If $\mathrm{cw}(G) \leq 2$ *(cograph), then* $\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil$*, and this bound is tight.*

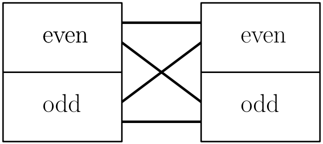
Every n-vertex graph G that admits a join satisfies $\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil$.



Theorem

If $\mathrm{cw}(G) \leq 2$ *(cograph), then* $\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil$*, and this bound is tight.*

Every n-vertex graph G that admits a join satisfies $\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil$.

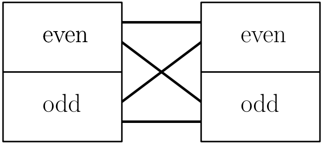


This bound is tight even for cographs:

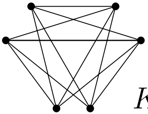
Theorem

If $\mathrm{cw}(G) \leq 2$ *(cograph), then* $\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil$*, and this bound is tight.*

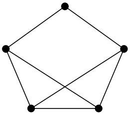
Every n-vertex graph G that admits a join satisfies $\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil$.



This bound is tight even for cographs:



$K_{2,2,2}$

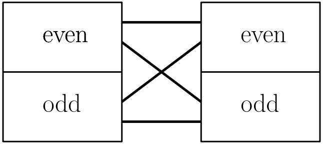


C_5^+

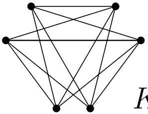
Theorem

If $\mathrm{cw}(G) \leq 2$ *(cograph), then* $\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil$*, and this bound is tight.*

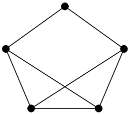
Every n-vertex graph G that admits a join satisfies $\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil$.



This bound is tight even for cographs:



$K_{2,2,2}$



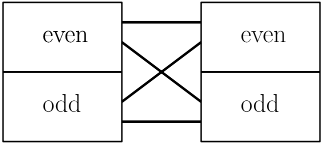
C_5^+

Odd graphs on four vertices: K_4, $K_{1,3}$, and $2K_2$.

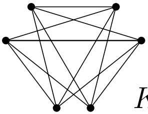
Theorem

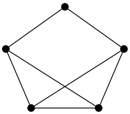
If $\mathsf{cw}(G) \leq 2$ *(cograph), then* $\mathsf{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil$*, and this bound is tight.*

Every n-vertex graph G that admits a join satisfies $\mathsf{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil$.



This bound is tight even for cographs:


$K_{2,2,2}$


C_5^+

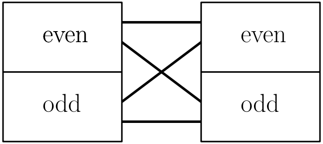
Odd graphs on four vertices: K_4, $K_{1,3}$, and $2K_2$.

Thus, $\mathsf{mos}(K_{2,2,2}) = \mathsf{mos}(C_5^+) = 2$

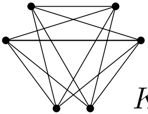
Theorem

If $\mathrm{cw}(G) \leq 2$ *(cograph), then* $\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil$*, and this bound is tight.*

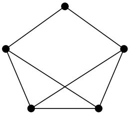
Every n-vertex graph G that admits a join satisfies $\mathrm{mos}(G) \geq 2 \cdot \left\lceil \frac{n-2}{4} \right\rceil$.



This bound is tight even for cographs:



$K_{2,2,2}$



C_5^+

Odd graphs on four vertices: K_4, $K_{1,3}$, and $2K_2$.

Thus, $\mathrm{mos}(K_{2,2,2}) = \mathrm{mos}(C_5^+) = 2 = 2 \cdot \left\lceil \frac{6-2}{4} \right\rceil = 2 \cdot \left\lceil \frac{5-2}{4} \right\rceil$.

Next section is...

1 Introduction

2 Our results

3 Some proofs

4 Further research

1. Algo in time $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$.

1. Algo in time $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$.
   Computing $\chi_{\mathrm{odd}}(G)$ parameterized by rw is FPT, W[1]-hard, XP?

1. Algo in time $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$.
   Computing $\chi_{\mathrm{odd}}(G)$ parameterized by rw is FPT, W[1]-hard, XP?
2. We proved that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1$.

1. Algo in time $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$.
   Computing $\chi_{\mathrm{odd}}(G)$ parameterized by rw is FPT, W[1]-hard, XP?
2. We proved that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1$.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G))$ for some f?

1. Algo in time $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$.
   Computing $\chi_{\mathrm{odd}}(G)$ parameterized by rw is FPT, W[1]-hard, XP?
2. We proved that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1$.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G))$ for some f? Would imply FPT algorithm.

1. Algo in time $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$.
   Computing $\chi_{\mathrm{odd}}(G)$ parameterized by rw is FPT, W[1]-hard, XP?
2. We proved that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1$.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G))$ for some f? Would imply FPT algorithm.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G)) \cdot \log n$ for some f?

1. Algo in time $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$.
   Computing $\chi_{\mathrm{odd}}(G)$ parameterized by rw is FPT, W[1]-hard, XP?
2. We proved that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1$.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G))$ for some f? Would imply FPT algorithm.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G)) \cdot \log n$ for some f? Would imply XP algorithm.

1. Algo in time $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$.
   Computing $\chi_{\mathrm{odd}}(G)$ parameterized by rw is FPT, W[1]-hard, XP?
2. We proved that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1$.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G))$ for some f? Would imply FPT algorithm.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G)) \cdot \log n$ for some f? Would imply XP algorithm.
3. The Chromatic Number problem is W[1]-hard param. by cw/rw.
   [Fomin, Golovach, Lokshtanov, Saurabh. 2010]

1. Algo in time $2^{\mathcal{O}(q \cdot \mathsf{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathsf{odd}}(G) \leq q$.
   Computing $\chi_{\mathsf{odd}}(G)$ parameterized by rw is FPT, W[1]-hard, XP?
2. We proved that $\chi_{\mathsf{odd}}(G) \leq \mathsf{tw}(G) + 1$.
   $\chi_{\mathsf{odd}}(G) \leq f(\mathsf{rw}(G))$ for some f? Would imply FPT algorithm.
   $\chi_{\mathsf{odd}}(G) \leq f(\mathsf{rw}(G)) \cdot \log n$ for some f? Would imply XP algorithm.
3. The Chromatic Number problem is W[1]-hard param. by cw/rw.
   [Fomin, Golovach, Lokshtanov, Saurabh. 2010]
   Can their reduction be adapted to computing $\chi_{\mathsf{odd}}(G)$?

1. Algo in time $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$.
   Computing $\chi_{\mathrm{odd}}(G)$ parameterized by rw is FPT, W[1]-hard, XP?
2. We proved that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1$.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G))$ for some f? Would imply FPT algorithm.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G)) \cdot \log n$ for some f? Would imply XP algorithm.
3. The Chromatic Number problem is W[1]-hard param. by cw/rw.
   [Fomin, Golovach, Lokshtanov, Saurabh. 2010]
   Can their reduction be adapted to computing $\chi_{\mathrm{odd}}(G)$?
4. Deciding whether $\chi_{\mathrm{odd}}(G) \leq q$ parameterized by tw:

1. Algo in time $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$.
   Computing $\chi_{\mathrm{odd}}(G)$ parameterized by rw is FPT, W[1]-hard, XP?

2. We proved that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1$.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G))$ for some f? Would imply FPT algorithm.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G)) \cdot \log n$ for some f? Would imply XP algorithm.

3. The CHROMATIC NUMBER problem is W[1]-hard param. by cw/rw.
   [Fomin, Golovach, Lokshtanov, Saurabh. 2010]
   Can their reduction be adapted to computing $\chi_{\mathrm{odd}}(G)$?

4. Deciding whether $\chi_{\mathrm{odd}}(G) \leq q$ parameterized by tw:
   - Natural DP algo in time $(2q)^{\mathrm{tw}} \cdot n^{\mathcal{O}(1)}$

1. Algo in time $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$.
   Computing $\chi_{\mathrm{odd}}(G)$ parameterized by rw is FPT, W[1]-hard, XP?
2. We proved that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1$.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G))$ for some f? Would imply FPT algorithm.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G)) \cdot \log n$ for some f? Would imply XP algorithm.
3. The Chromatic Number problem is W[1]-hard param. by cw/rw.
   [Fomin, Golovach, Lokshtanov, Saurabh. 2010]
   Can their reduction be adapted to computing $\chi_{\mathrm{odd}}(G)$?
4. Deciding whether $\chi_{\mathrm{odd}}(G) \leq q$ parameterized by tw:
   - Natural DP algo in time $(2q)^{\mathrm{tw}} \cdot n^{\mathcal{O}(1)} \leq (2\mathrm{tw} + 2)^{\mathrm{tw}} \cdot n^{\mathcal{O}(1)}$.

1. Algo in time $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$.
   Computing $\chi_{\mathrm{odd}}(G)$ parameterized by rw is FPT, W[1]-hard, XP?
2. We proved that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1$.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G))$ for some f? Would imply FPT algorithm.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G)) \cdot \log n$ for some f? Would imply XP algorithm.
3. The CHROMATIC NUMBER problem is W[1]-hard param. by cw/rw.
   [Fomin, Golovach, Lokshtanov, Saurabh. 2010]
   Can their reduction be adapted to computing $\chi_{\mathrm{odd}}(G)$?
4. Deciding whether $\chi_{\mathrm{odd}}(G) \leq q$ parameterized by tw:
   - Natural DP algo in time $(2q)^{\mathrm{tw}} \cdot n^{\mathcal{O}(1)} \leq (2\mathrm{tw}+2)^{\mathrm{tw}} \cdot n^{\mathcal{O}(1)}$.
   - It can be proved that $\nexists$ $\mathrm{tw}^{o(\mathrm{tw})} \cdot n^{\mathcal{O}(1)}$ under the ETH ✓

1. Algo in time $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$.
   Computing $\chi_{\mathrm{odd}}(G)$ parameterized by rw is FPT, W[1]-hard, XP?

2. We proved that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1$.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G))$ for some f? Would imply FPT algorithm.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G)) \cdot \log n$ for some f? Would imply XP algorithm.

3. The Chromatic Number problem is W[1]-hard param. by cw/rw.
   [Fomin, Golovach, Lokshtanov, Saurabh. 2010]
   Can their reduction be adapted to computing $\chi_{\mathrm{odd}}(G)$?

4. Deciding whether $\chi_{\mathrm{odd}}(G) \leq q$ parameterized by tw:
   - Natural DP algo in time $(2q)^{\mathrm{tw}} \cdot n^{\mathcal{O}(1)} \leq (2\mathrm{tw}+2)^{\mathrm{tw}} \cdot n^{\mathcal{O}(1)}$.
   - It can be proved that $\nexists\ \mathrm{tw}^{o(\mathrm{tw})} \cdot n^{\mathcal{O}(1)}$ under the ETH ✓
   - Right constants under the SETH?

1. Algo in time $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$.
   Computing $\chi_{\mathrm{odd}}(G)$ parameterized by rw is FPT, W[1]-hard, XP?

2. We proved that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1$.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G))$ for some f? Would imply FPT algorithm.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G)) \cdot \log n$ for some f? Would imply XP algorithm.

3. The Chromatic Number problem is W[1]-hard param. by cw/rw.
   [Fomin, Golovach, Lokshtanov, Saurabh. 2010]
   Can their reduction be adapted to computing $\chi_{\mathrm{odd}}(G)$?

4. Deciding whether $\chi_{\mathrm{odd}}(G) \leq q$ parameterized by tw:
   - Natural DP algo in time $(2q)^{\mathrm{tw}} \cdot n^{\mathcal{O}(1)} \leq (2\mathrm{tw} + 2)^{\mathrm{tw}} \cdot n^{\mathcal{O}(1)}$.
   - It can be proved that $\nexists\ \mathrm{tw}^{o(\mathrm{tw})} \cdot n^{\mathcal{O}(1)}$ under the ETH ✓
   - Right constants under the SETH?

5. We know $\mathrm{mes}(G) \geq n/2$.

1. Algo in time $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$.
   Computing $\chi_{\mathrm{odd}}(G)$ parameterized by rw is FPT, W[1]-hard, XP?
2. We proved that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1$.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G))$ for some f? Would imply FPT algorithm.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G)) \cdot \log n$ for some f? Would imply XP algorithm.
3. The Chromatic Number problem is W[1]-hard param. by cw/rw.
   [Fomin, Golovach, Lokshtanov, Saurabh. 2010]
   Can their reduction be adapted to computing $\chi_{\mathrm{odd}}(G)$?
4. Deciding whether $\chi_{\mathrm{odd}}(G) \leq q$ parameterized by tw:
   - Natural DP algo in time $(2q)^{\mathrm{tw}} \cdot n^{\mathcal{O}(1)} \leq (2\mathrm{tw} + 2)^{\mathrm{tw}} \cdot n^{\mathcal{O}(1)}$.
   - It can be proved that $\nexists$ $\mathrm{tw}^{o(\mathrm{tw})} \cdot n^{\mathcal{O}(1)}$ under the ETH ✓
   - Right constants under the SETH?
5. We know $\mathrm{mes}(G) \geq n/2$. Deciding $\mathrm{mes}(G) \geq n/2 + k$ with param. k?

1. Algo in time $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$.
   Computing $\chi_{\mathrm{odd}}(G)$ parameterized by rw is FPT, W[1]-hard, XP?

2. We proved that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1$.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G))$ for some f? Would imply FPT algorithm.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G)) \cdot \log n$ for some f? Would imply XP algorithm.

3. The Chromatic Number problem is W[1]-hard param. by cw/rw.
   [Fomin, Golovach, Lokshtanov, Saurabh. 2010]
   Can their reduction be adapted to computing $\chi_{\mathrm{odd}}(G)$?

4. Deciding whether $\chi_{\mathrm{odd}}(G) \leq q$ parameterized by tw:
   - Natural DP algo in time $(2q)^{\mathrm{tw}} \cdot n^{\mathcal{O}(1)} \leq (2\mathrm{tw}+2)^{\mathrm{tw}} \cdot n^{\mathcal{O}(1)}$.
   - It can be proved that $\nexists\ \mathrm{tw}^{o(\mathrm{tw})} \cdot n^{\mathcal{O}(1)}$ under the ETH ✓
   - Right constants under the SETH?

5. We know $\mathrm{mes}(G) \geq n/2$. Deciding $\mathrm{mes}(G) \geq n/2 + k$ with param. k?

6. The problems that we considered can be seen as the "parity version" of Independent Set and q-Coloring.

1. Algo in time $2^{\mathcal{O}(q \cdot \mathrm{rw})} \cdot n^{\mathcal{O}(1)}$ for deciding whether $\chi_{\mathrm{odd}}(G) \leq q$.
   Computing $\chi_{\mathrm{odd}}(G)$ parameterized by rw is FPT, W[1]-hard, XP?

2. We proved that $\chi_{\mathrm{odd}}(G) \leq \mathrm{tw}(G) + 1$.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G))$ for some f? Would imply FPT algorithm.
   $\chi_{\mathrm{odd}}(G) \leq f(\mathrm{rw}(G)) \cdot \log n$ for some f? Would imply XP algorithm.

3. The CHROMATIC NUMBER problem is W[1]-hard param. by cw/rw.
   [Fomin, Golovach, Lokshtanov, Saurabh. 2010]
   Can their reduction be adapted to computing $\chi_{\mathrm{odd}}(G)$?

4. Deciding whether $\chi_{\mathrm{odd}}(G) \leq q$ parameterized by tw:
   - Natural DP algo in time $(2q)^{\mathrm{tw}} \cdot n^{\mathcal{O}(1)} \leq (2\mathrm{tw} + 2)^{\mathrm{tw}} \cdot n^{\mathcal{O}(1)}$.
   - It can be proved that $\nexists\ \mathrm{tw}^{o(\mathrm{tw})} \cdot n^{\mathcal{O}(1)}$ under the ETH ✓
   - Right constants under the SETH?

5. We know $\mathrm{mes}(G) \geq n/2$. Deciding $\mathrm{mes}(G) \geq n/2 + k$ with param. k?

6. The problems that we considered can be seen as the "parity version" of INDEPENDENT SET and q-COLORING. Other problems?

Gràcies!